Collective behavior in nuclei

Signatures of phase transitional behavior

An enhanced link between nuclear masses and structure

2-nucleon transfer reactions: a measure of structural change

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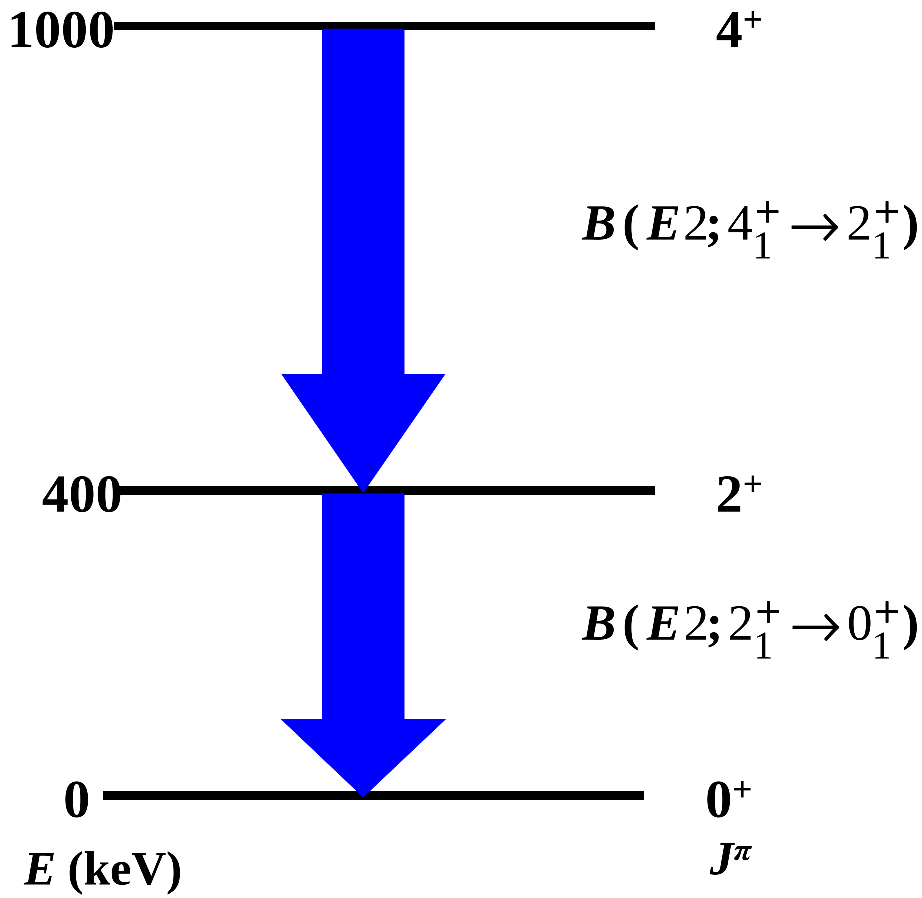
GANIL, Nov. 21, 2008

Simple Observables - Even-Even Nuclei

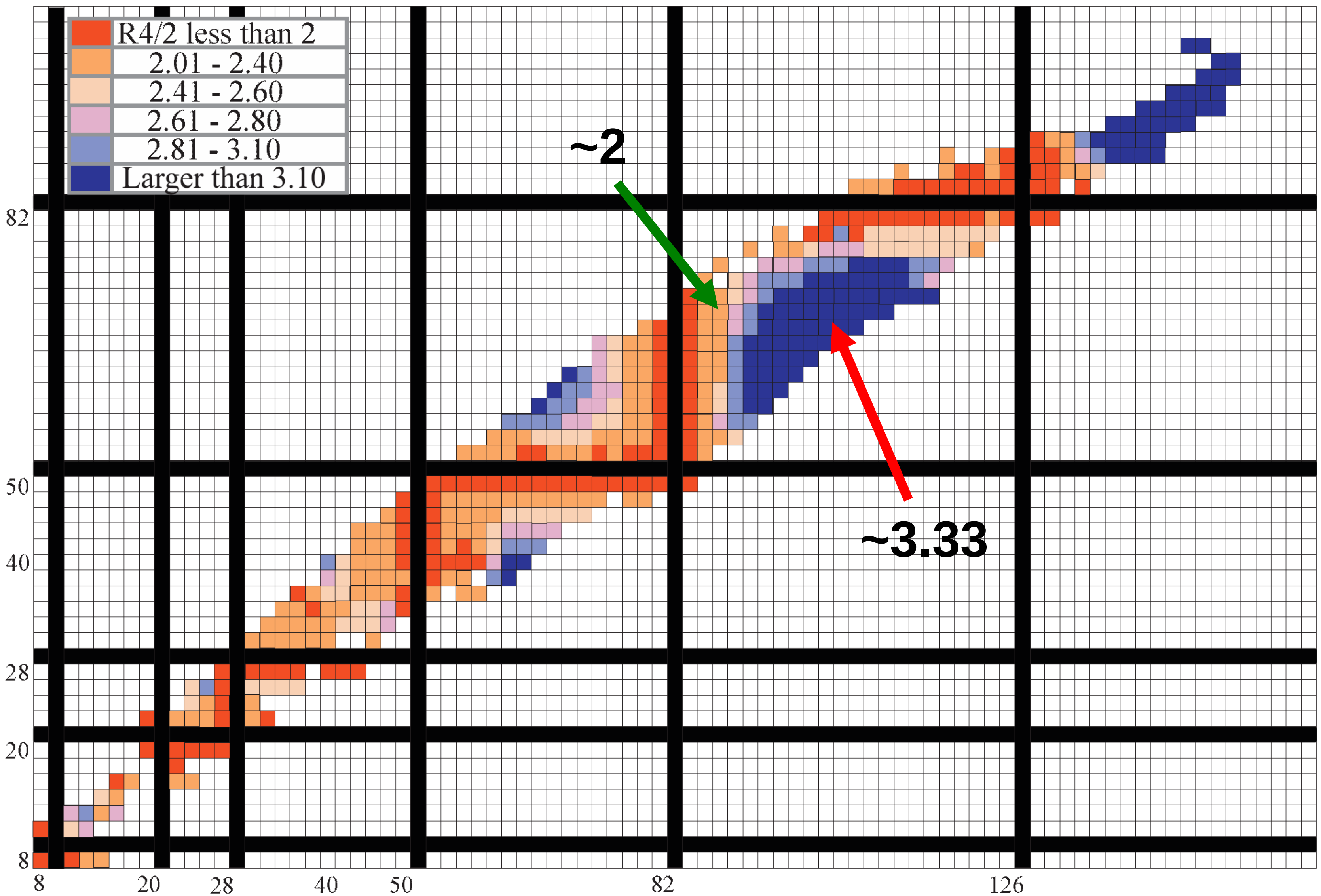


$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

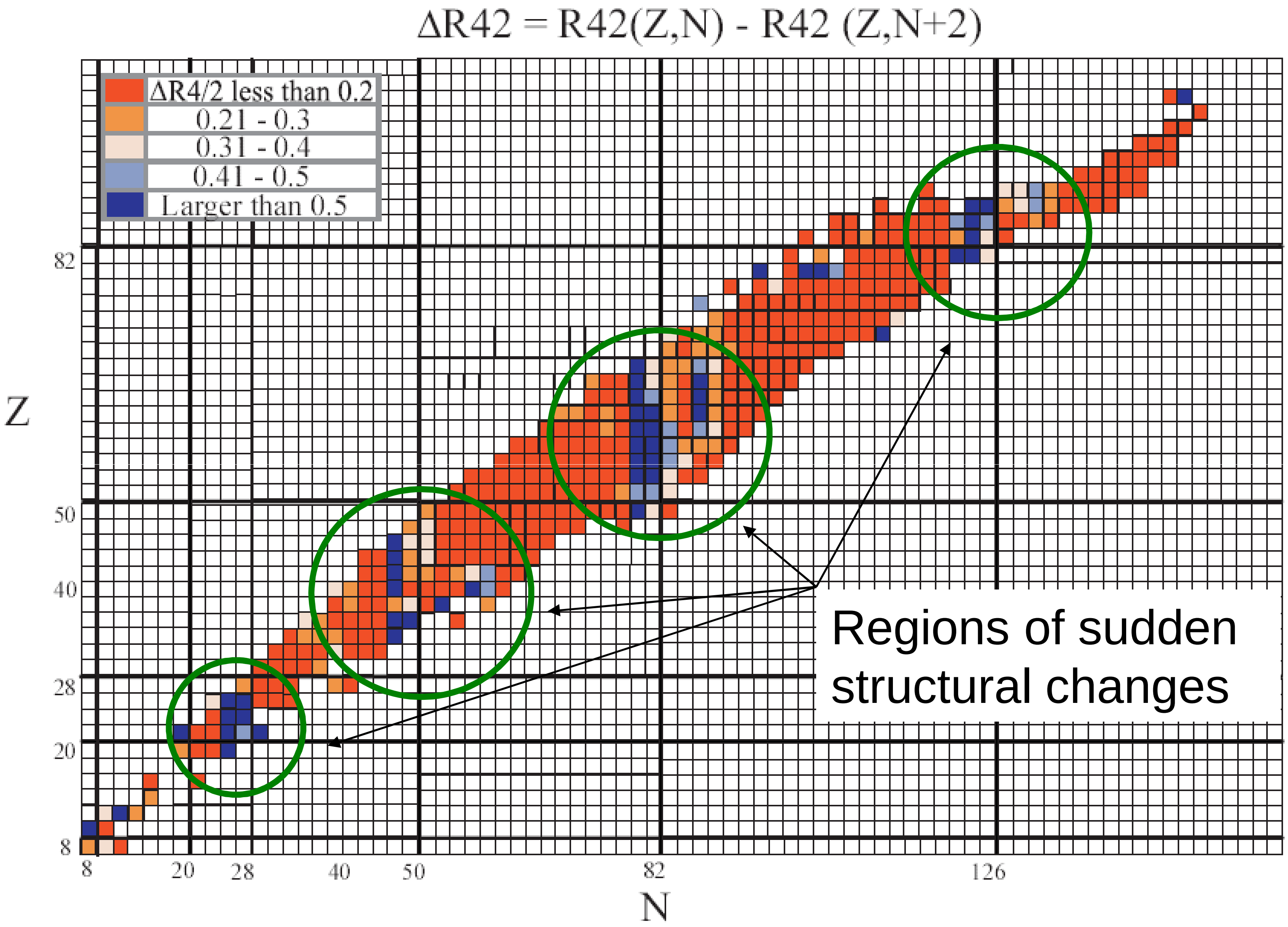
Vibrator: $R_{4/2} = 2$
Rotor: $R_{4/2} = 3.33$

Masses, σ(p,t)

Broad perspective on structural evolution: $R_{4/2}$

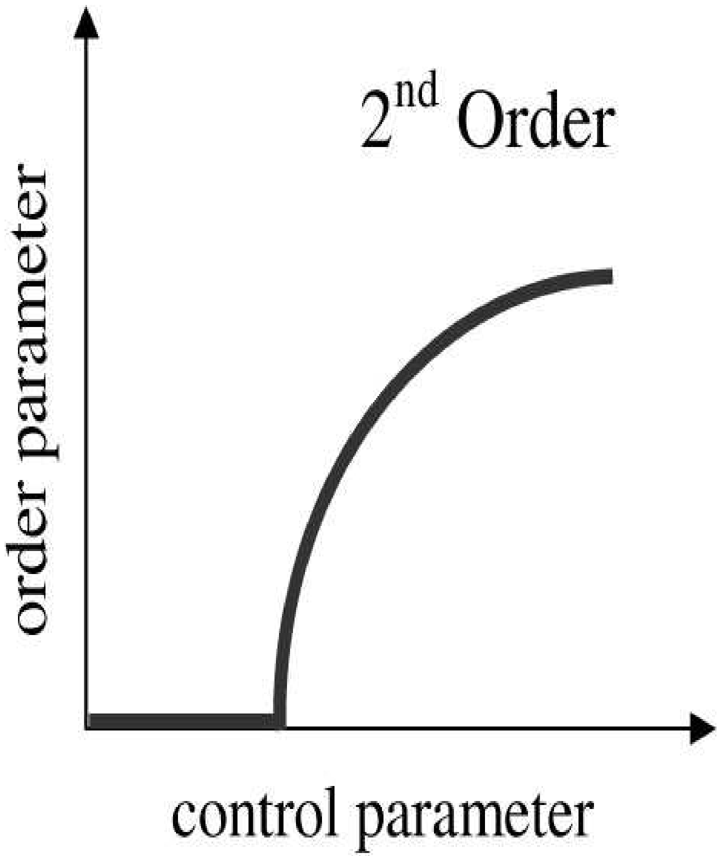


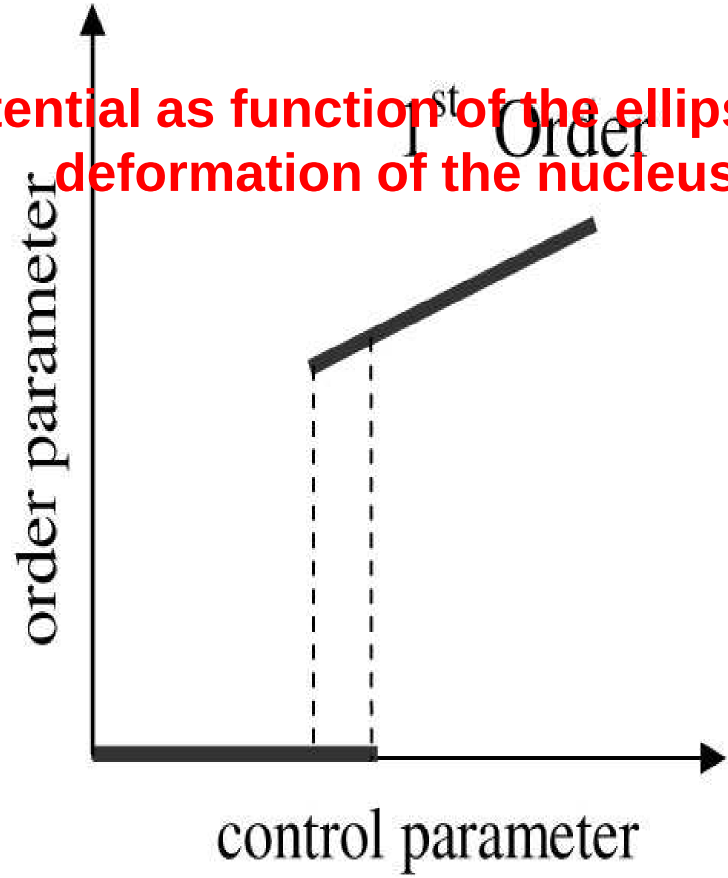
Where do we find shape/phase transitional behavior?



From Burcu Cakirli

Quantum phase transitions in equilibrium shapes of nuclei with *N*, *Z*





Potential as function of the ellipsoidal deformation of the nucleus

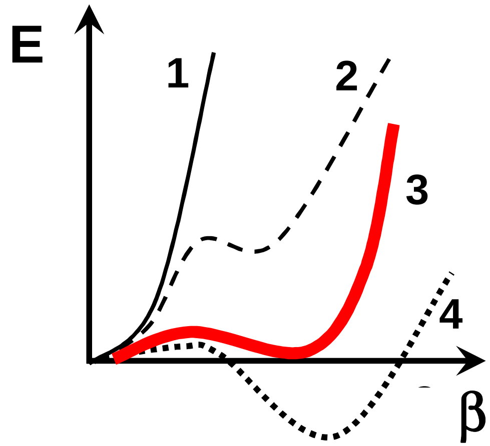
For nuclear shape phase transitions the control parameter is nucleon number

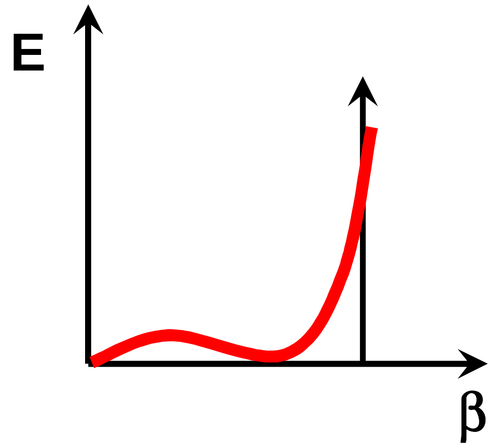
Critical Point Symmetries

X(5)

First Order Phase Transition – Phase Coexistence

Energy surface changes with valence nucleon number





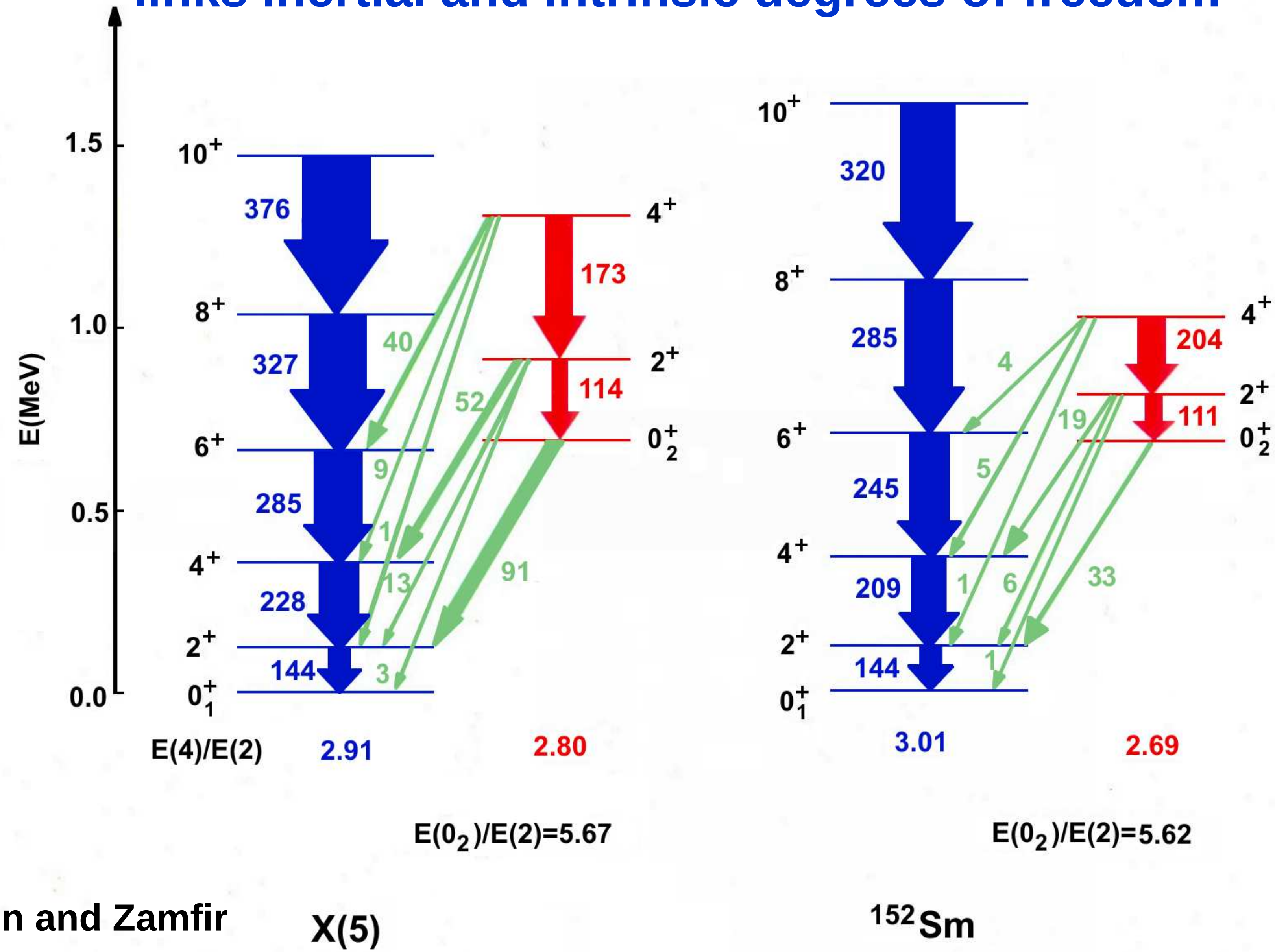
Bessel equation

$$\tilde{\xi}'' + \frac{\tilde{\xi}'}{z} + \left[1 - \frac{v^2}{z^2}\right]\tilde{\xi} = 0; \qquad \tilde{\xi}(\beta_w) = 0.$$

$$v = \left(\frac{L(L+1)}{3} + \frac{9}{4}\right)^{1/2}$$

Iachello

Critical Point Symmetry, X(5)
Parameter-free (except for scale):
links inertial and intrinsic degrees of freedom



Casten and Zamfir

Is there a signature that –

a) distinguishes first and second order phase transitions;

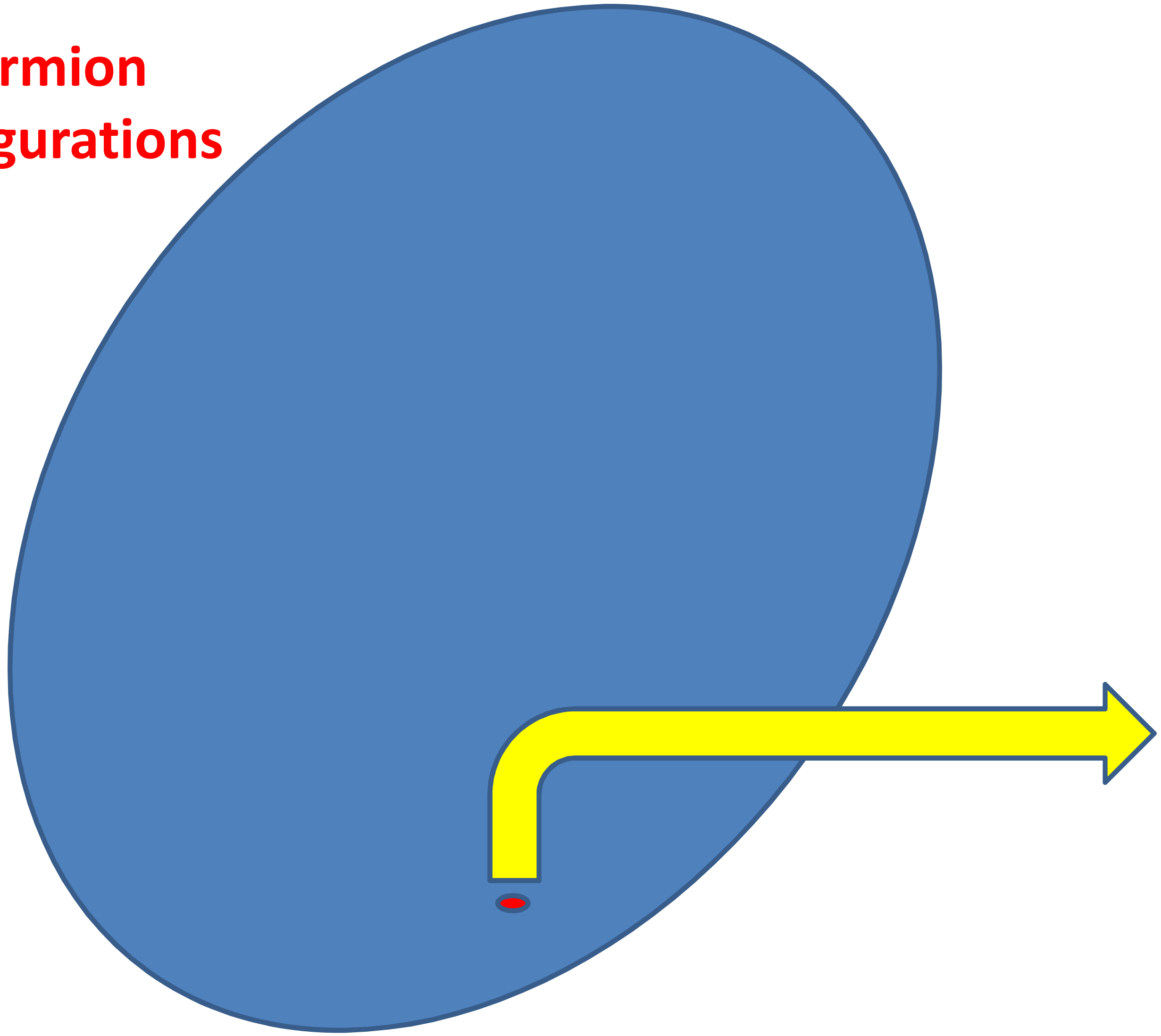
b) is robust with respect to valence nucleon number;

c) is based on simple-to-observe data?

To study this, we use the IBA model since it spans a wide range of collective structures and, in the large valence nucleon number limit, can be used to simulate geometrical models. First the briefest of introductions to the model.

Shell Model Configurations

Fermion configurations



Boson configurations

(by considering only configurations of pairs of fermions with J = 0 or 2, called s and d bosons.) Huge truncation of the Shell Model.

Dynamical Symmetries and Phase Transitions

Vibrator

I. U(6) ⊃ U(5) ⊃ O(5) ⊃ O(3) U(5)

N n_d ■ n J

Rotor

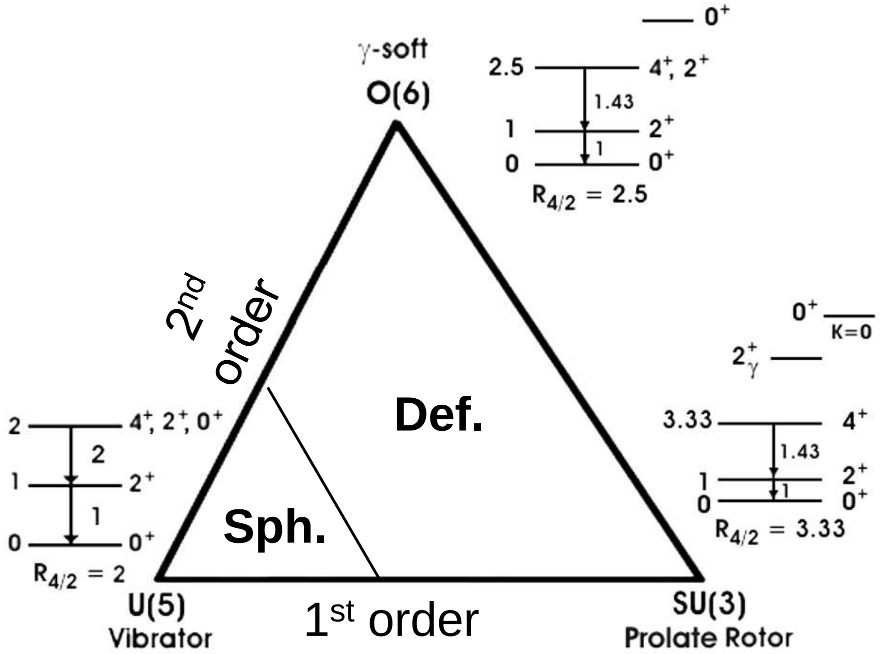
II. U(6) ⊃ SU(3) ⊃ O(3) SU(3)

N (●, ❍) K J

Gamma-soft rotor

III. U(6) ⊃ O(6) ⊃ O(5) ⊃ O(3) O(6)

N σ ♦ ■ J



The IBA: convenient model that spans the entire triangle of colllective structures

Sph. Driving — Def. Driving

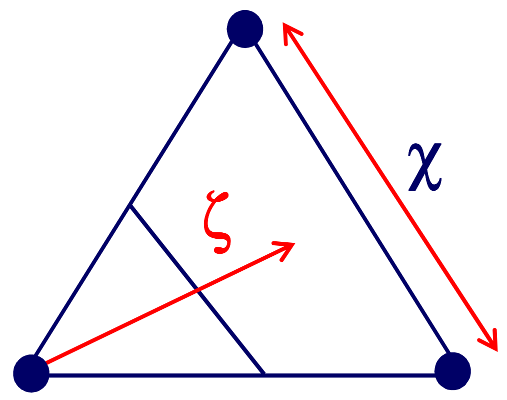
$$H = \varepsilon\, n_d \; - \; \kappa\, Q \cdot Q$$

Parameters: κ/ε , χ (within Q)

Competition between competing symmetries, determined by κ/ε
However, awkward that κ/ε varies from 0 to infinity
Replace by more convenient form:

$$H = c\left[(1 - \zeta)\, n_d - \frac{\zeta}{4N_B} Q^{\chi} \cdot Q^{\chi} \right]$$

Span triangle with ζ and χ



Empirical signature of 1st and 2nd order

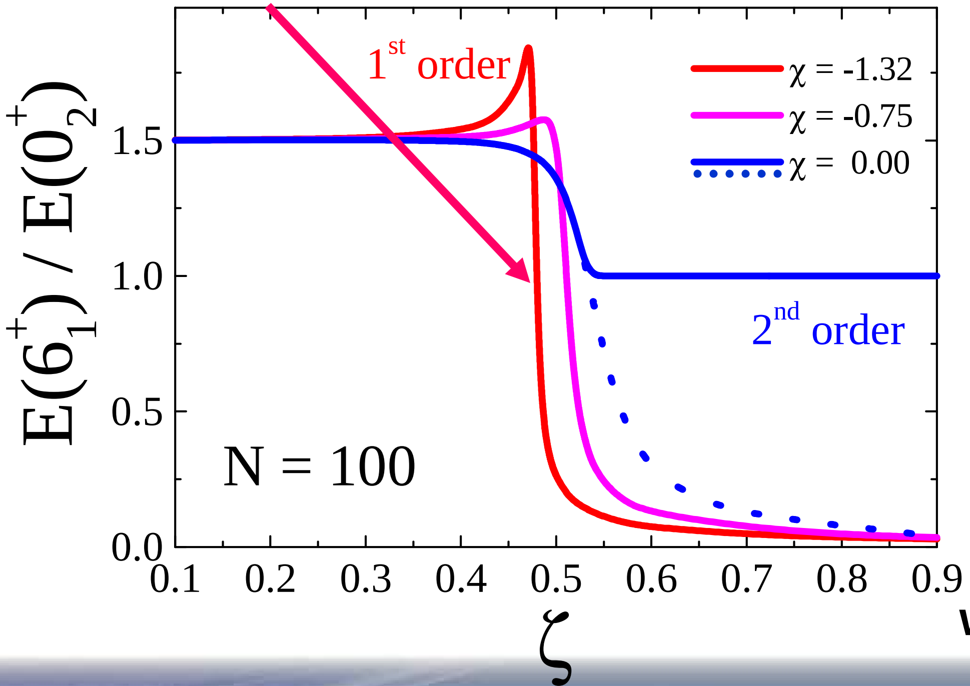
Energy ratio between 6$^+$ of ground state and first excited 0$^+$

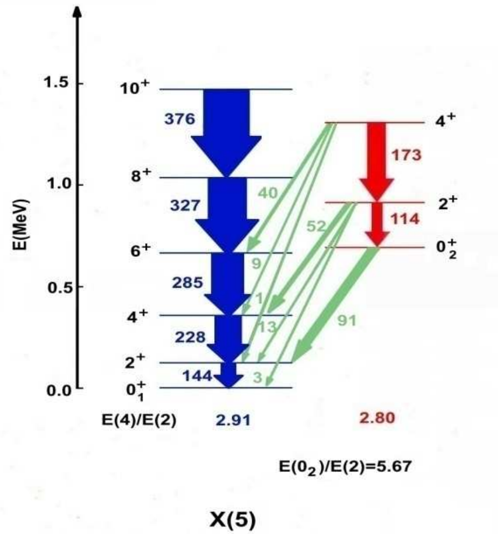
$$\frac{E(6_1^+)}{E(0_2^+)} = \begin{cases} 1.5 & U(5) \\ \rightarrow 0 & SU(3) \end{cases}$$

Vibrator

Rotor

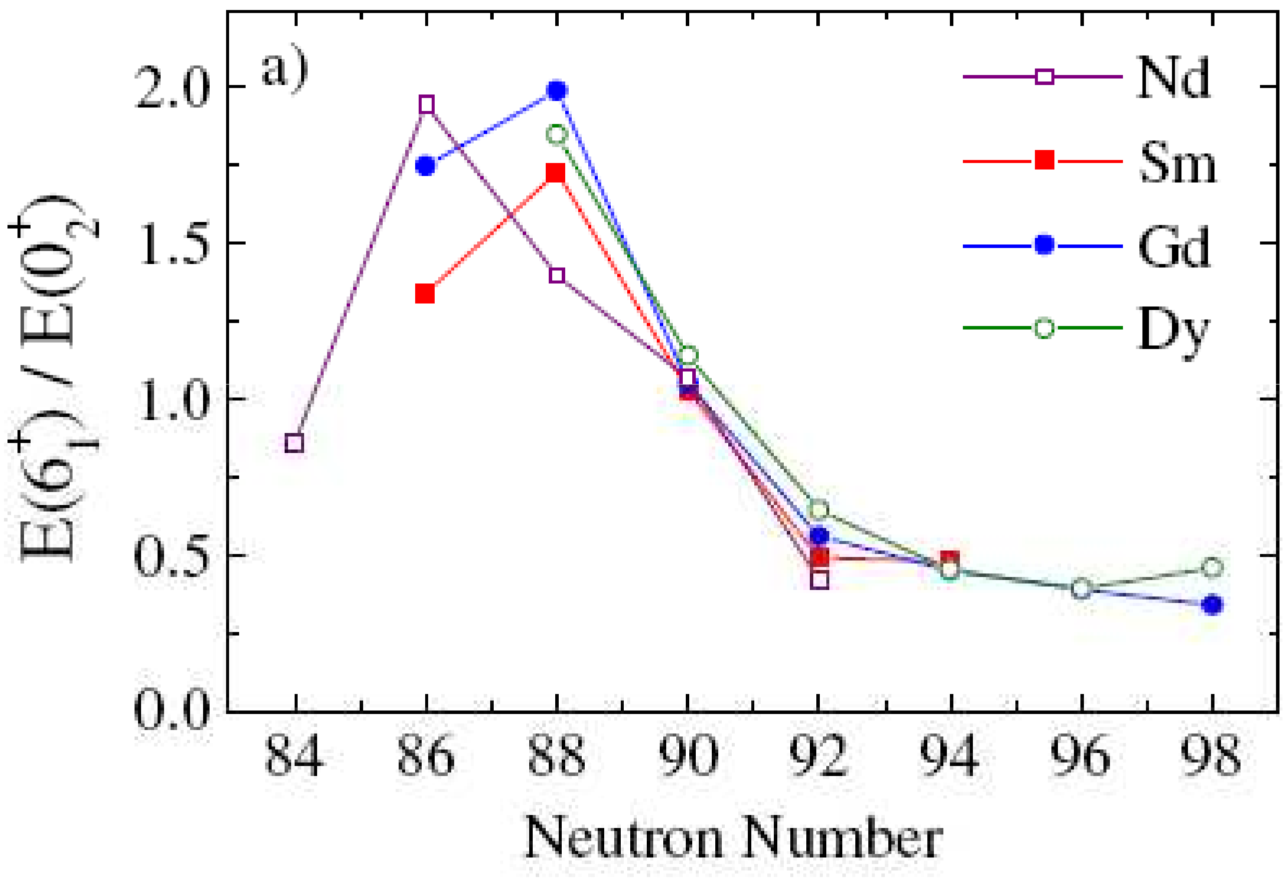
~1 at Ph. Tr ~ X(5)



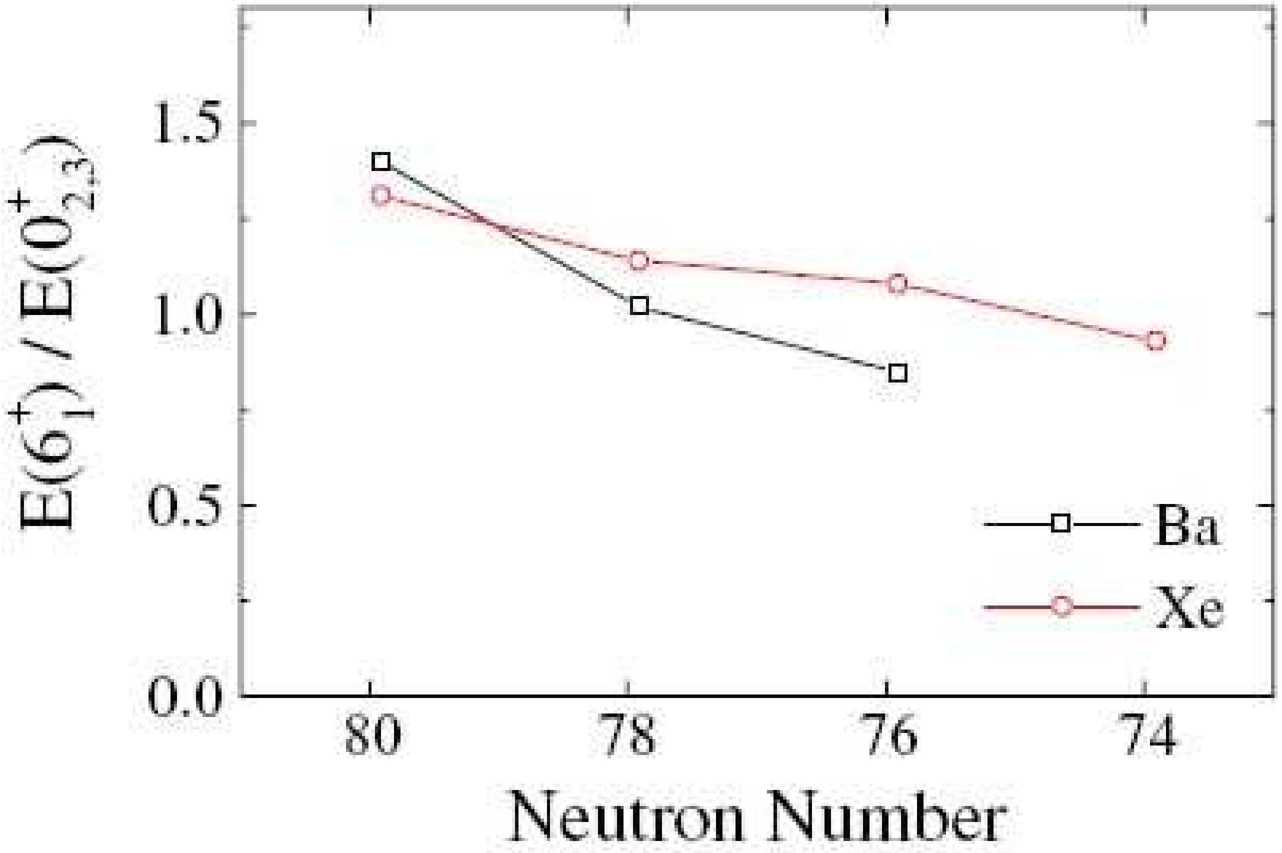


w/ Bonatsos and McCutchan

First Order Phase Transition



Second Order Phase Transition



And now for something completely different (sort of).

Binding energies

A heretofore unrecognized, greatly enhanced, sensitivity to structure

w/ Cakirli

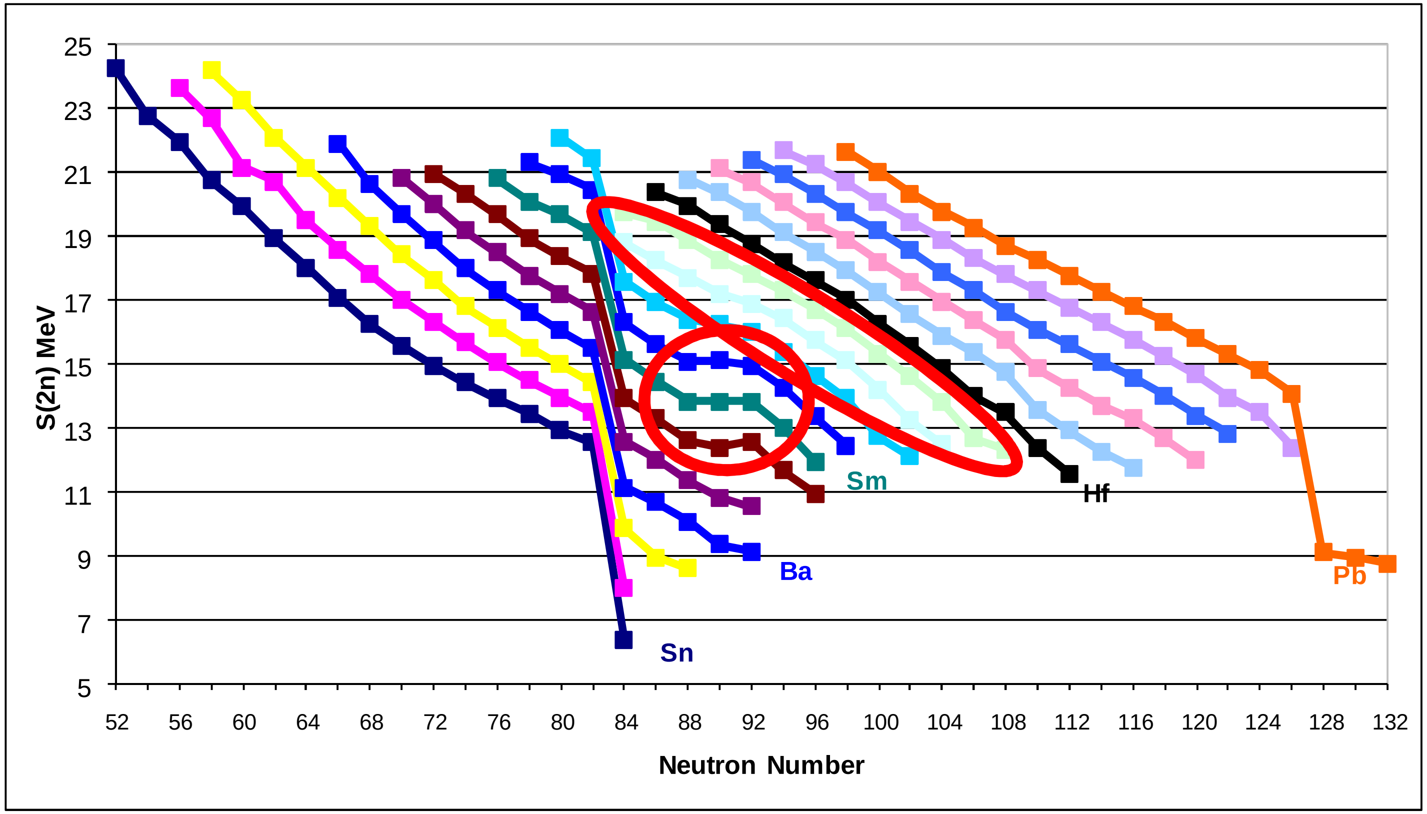
Two-neutron separation energies

$$S_{2n} = A + BN + S_{2n}(\text{Coll.})$$

Normal behavior: linear segments with breaks after closed shells

Discontinuities at first order phase transitions

Curvature in isotopic chains



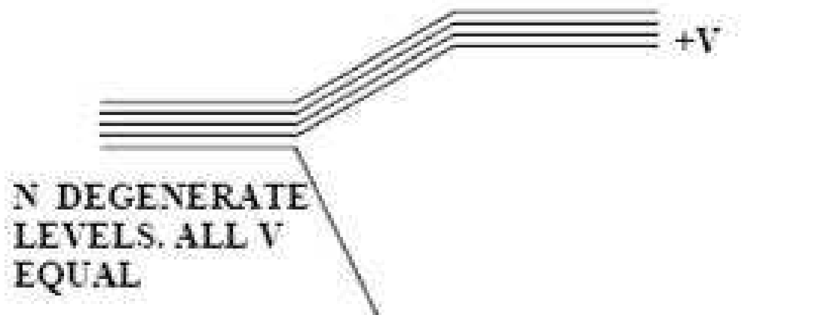
We can use any collective model to calculate the collective contributions to S_{2n}.

IBA for binding/separation energies to calculate S_{2n}(Coll.)

(Note: we could also use the IBA to calculate the A + BN part of S_{2n} by adding terms to the IBA Hamiltonian corresponding to the Casimir operators of U(6) but that is not necessary.)

- **The same coherence that accounts for the spectroscopy of collective levels and transitions also enables the study of masses, binding energies, and separation energies with the IBA, often in a paramter-free way.**
- **Consider as an example, several chains of deformed rare earth nuclei.**

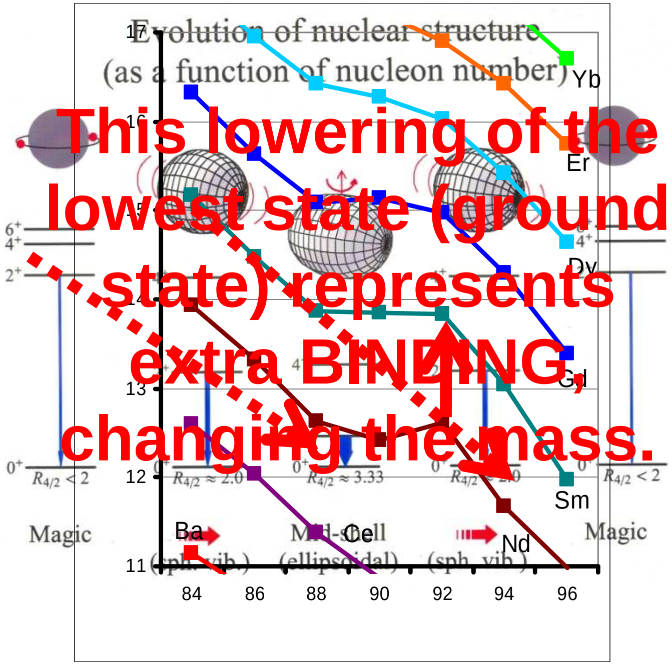
Collectivity – origin and consequences



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}[\phi_1 + \phi_2 + \ldots + \phi_N]$$

Also crucial for masses

Crucial for structure

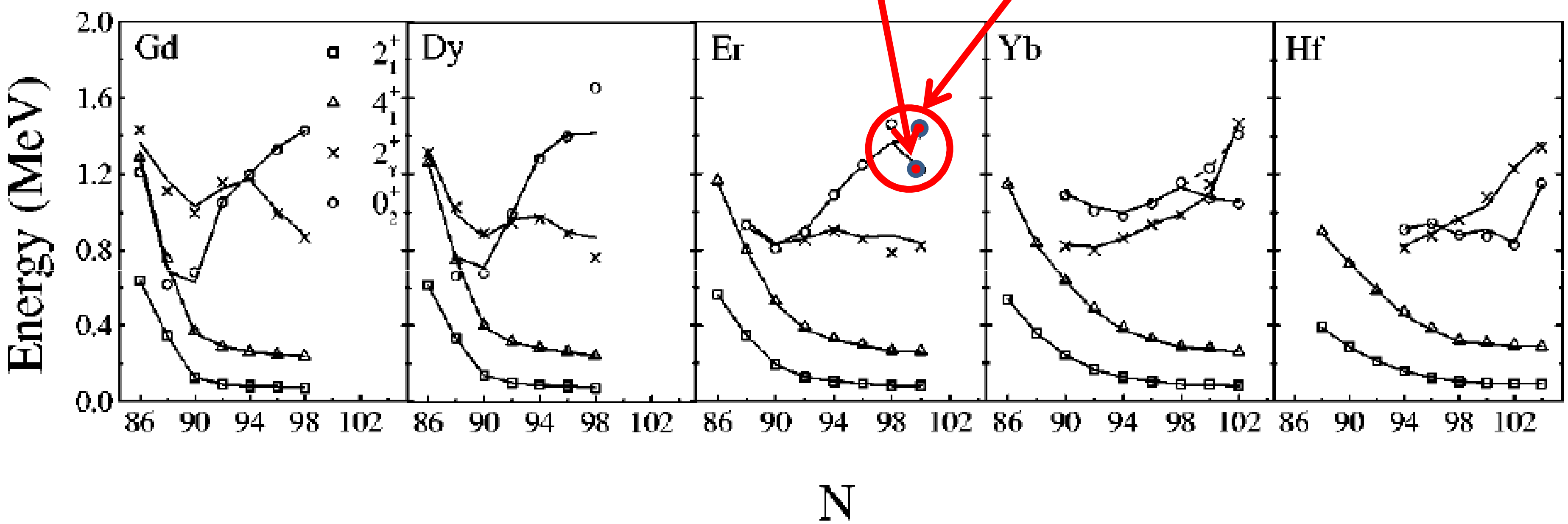


This lowering of the lowest state (ground state) represents extra BINDING, changing the mass.

Which 0+ level is collective and which is a 2-quasi-particle state?

Evolution of level energies in rare earth nuclei: Fit levels, B(E2) values, then calculate BE's

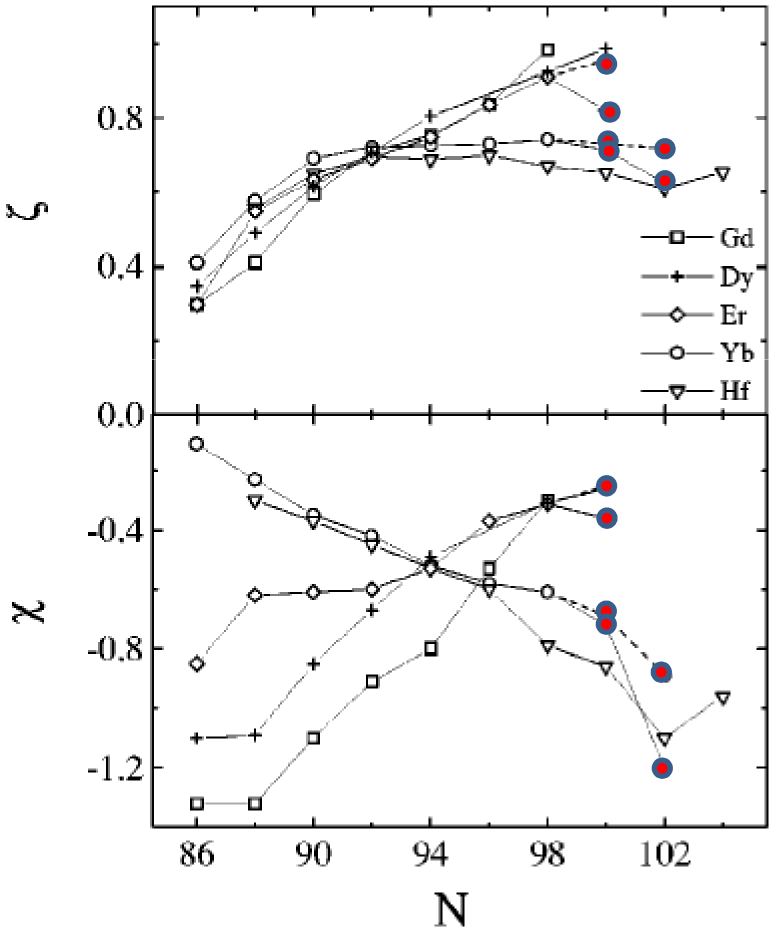
But note:



So, can do alternate collective model fits, assuming one or the other state is the collective one. Look at implications for masses.

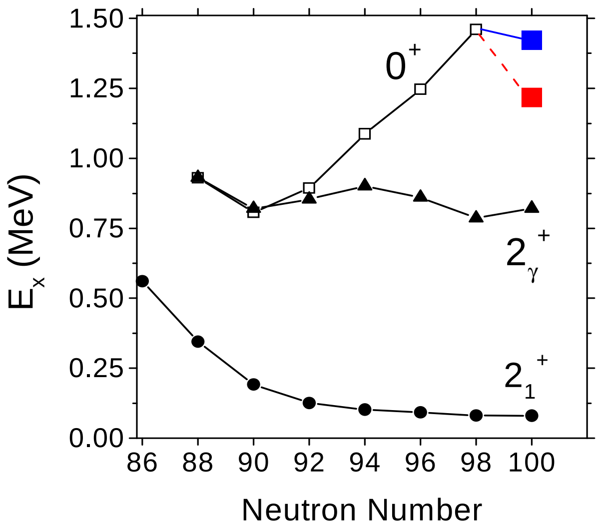
How much will the calculated binding energies change for these two fits – to the 0^+ states at 1222 and 1422 keV?

$$H = c\left[(1-\zeta)\, n_d - \frac{\zeta}{4N_B}\, Q^{\chi}\cdot Q^{\chi}\right]$$



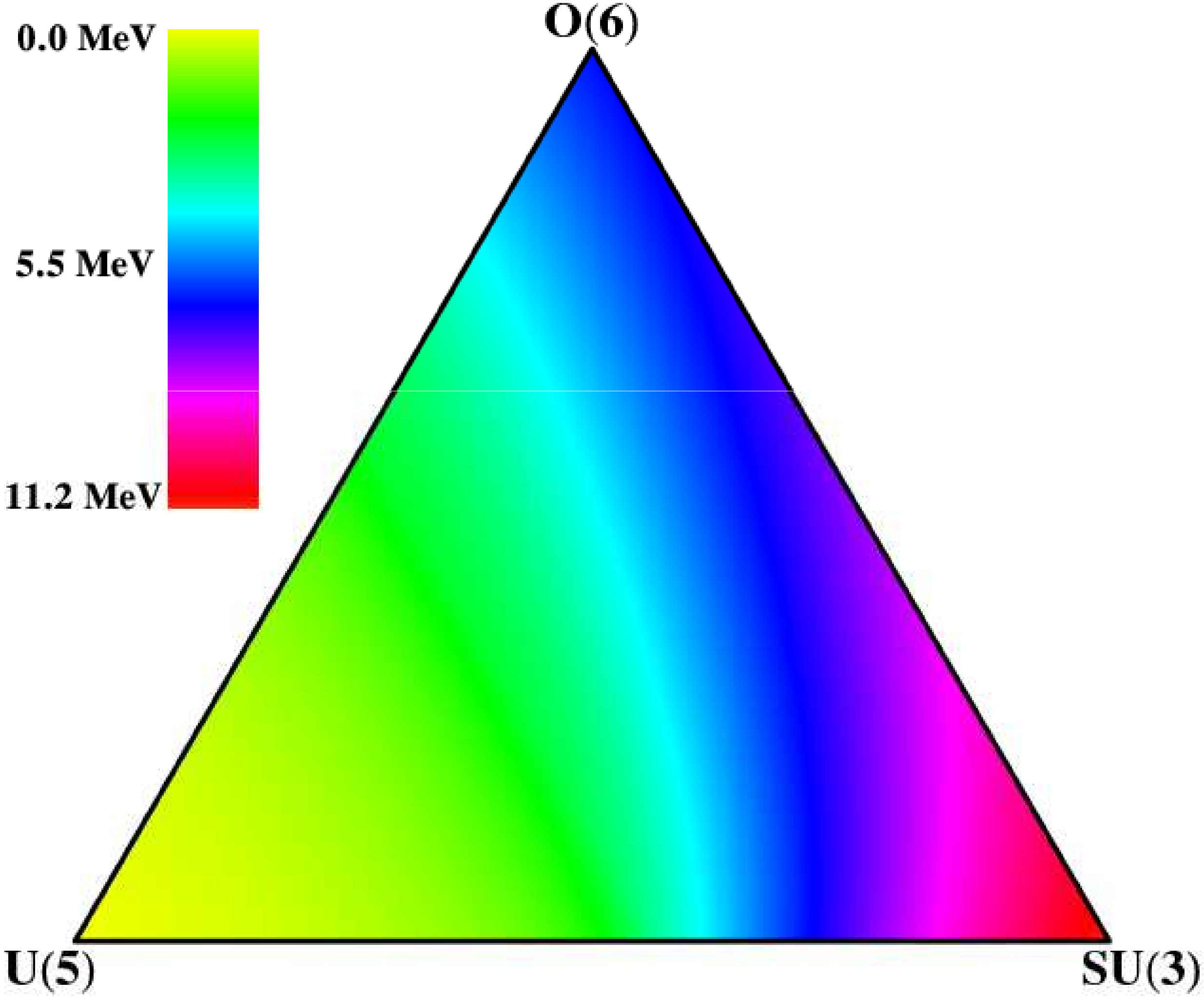
Now, lets look at the calculated binding energies (S_{2n} values) with these different sets of parameters chosen to fit different excited states.

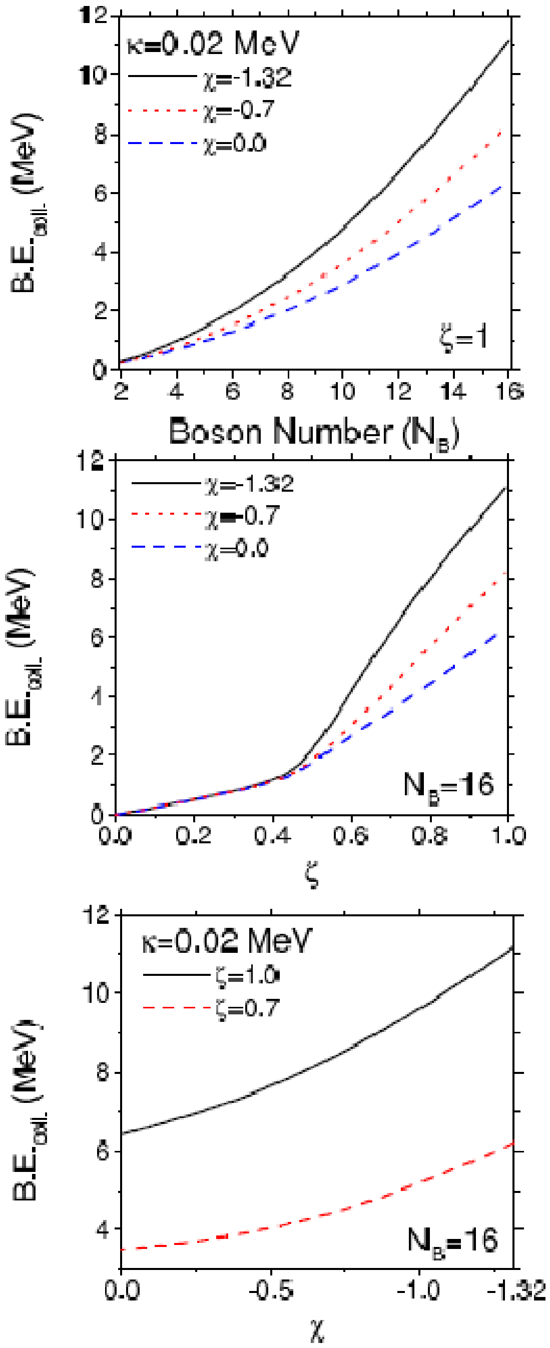
!!!

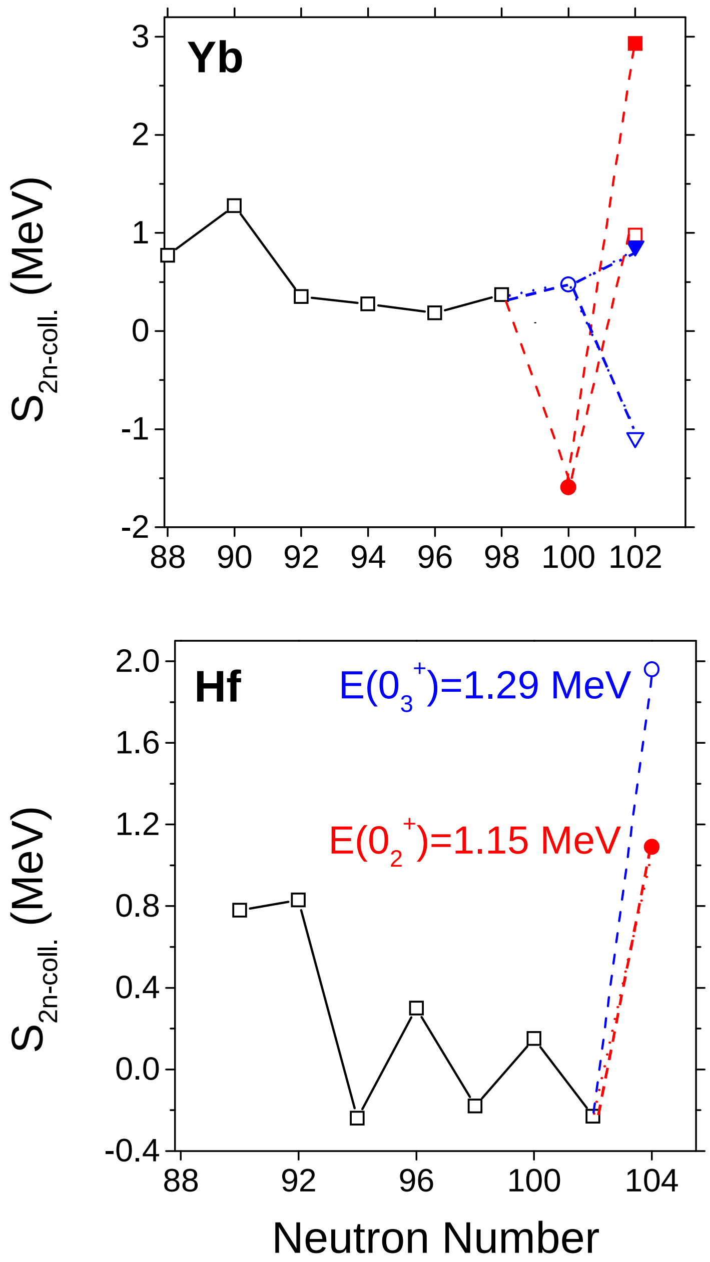


Note: These two levels differ by 200 keV

IBA-1 Binding Energies, $N_B = 16$




κ=0.02 MeV
χ=-1.32
χ=-0.7
χ=0.0
ζ=1
B.E.coll. (MeV)
Boson Number (NB)
χ=-1.32
χ=-0.7
χ=0.0
NB=16
B.E.coll. (MeV)
ζ
κ=0.02 MeV
ζ=1.0
ζ=0.7
NB=16
B.E.coll. (MeV)
χ



Yb
Hf
$E(0_3^+)$=1.29 MeV
$E(0_2^+)$=1.15 MeV
$S_{2n\text{-coll.}}$ (MeV)
Neutron Number

Binding energies and structure

Conclusions (soft version)

These results show a link between masses and structure that is much more sensitive than heretofore realized. This is a new realization and needs much further study.

Conclusions (strong version)

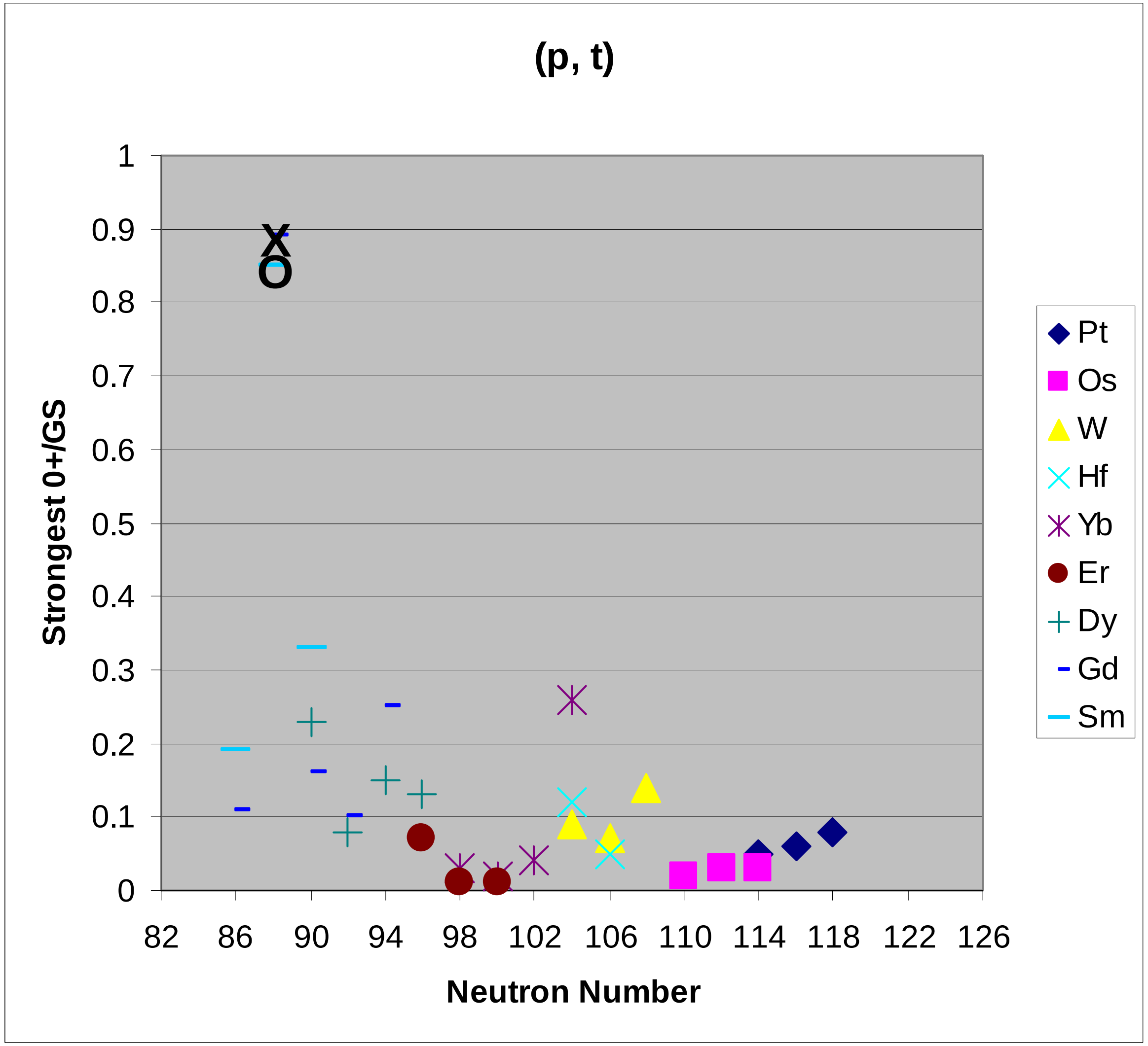
- No one can ever again do a structure calculation without looking at the implications for masses.
- Anyone who measures a mass should check to see what implications it has for understanding the structure of the nucleus (e.g., can the mass tell us which excited 0^+ state is collective ???!!!!).

Two-Nucleon Transfer Reactions

A new interpretation

A unified approach to the diverse phenomenology

Empirical survey of (p,t) reaction strengths to **0** $^{+}$ states



Nearly always: cross sections to excited 0^{+} states are a small percentage of the ground state cross section.

In the spherical – deformed transition region at N = 90, excited state cross sections are comparable to those of the ground state.

The "standard interpretation" (since ca. 1960s) of 2-nucleon transfer reactions to 0^+ states in collective nuclei

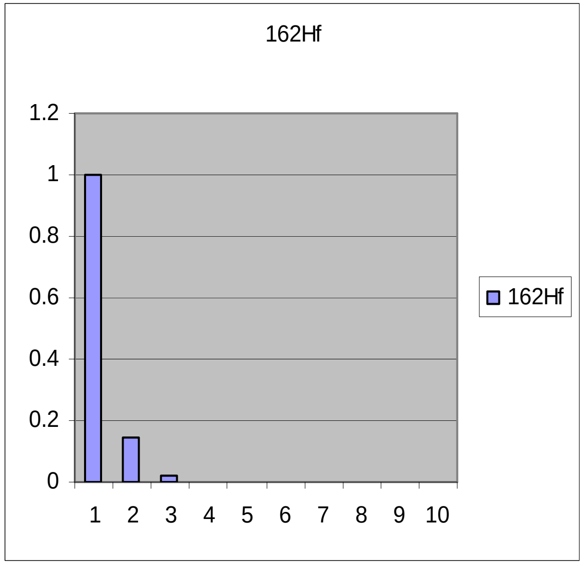
- **Most nuclei**: Cross sections to excited 0^+ states are small because the collective components add coherently for the ground state but cancel for the orthogonal excited states.

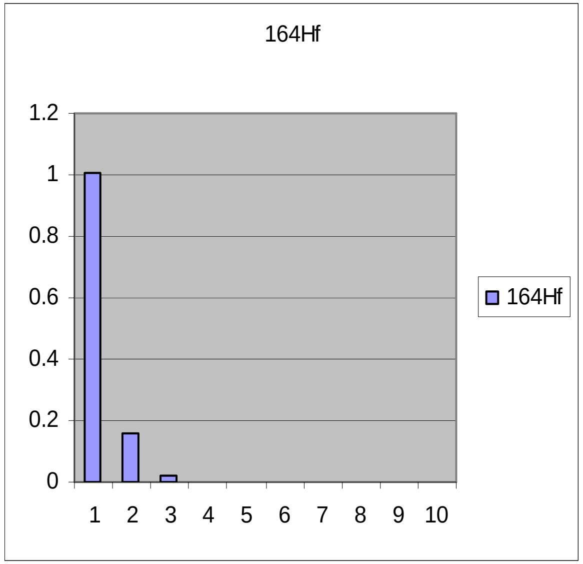
- **Phase transition region**: Spherical and deformed states coexist and mix. Hence a reaction such as (p,t) on a deformed ^{156}Gd target populates both the "quasi-deformed" ground and "quasi-spherical" excited states of ^{154}Gd

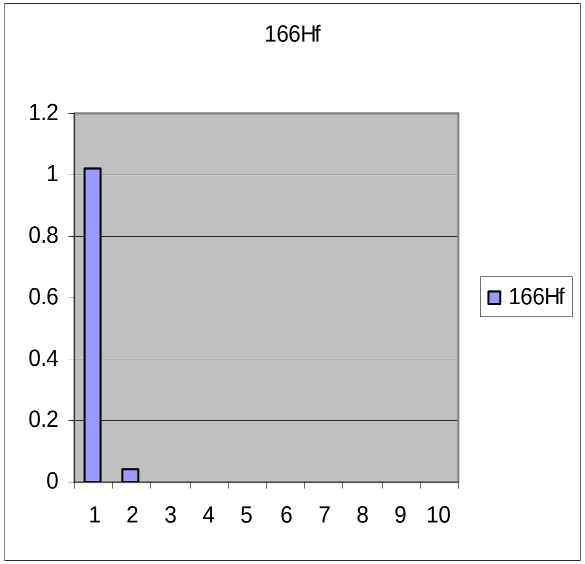
IBA calculations of (p,t) and (t,p)

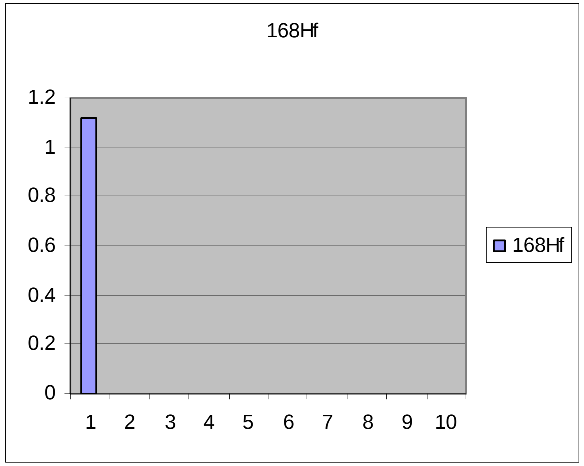
IBA is well-suited to study these because it embodies both a wide range of collective structures and, being based on s, and d bosons, naturally contains an appropriate operator for these reactions which are just s-boson transfers

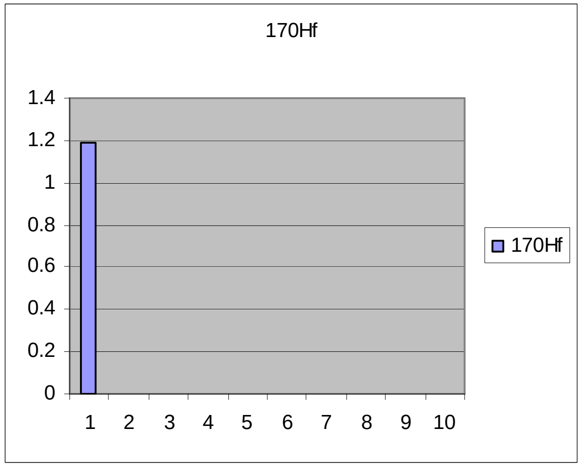
- **Calculate IBA for target and final nuclei. For the rare earth region, one can do this with existing parameter sets.**
- **Calculate the 2-nucleon transfer cross sections relative to the ground state so these calculations are parameter-free.**
- **Compare with data – excellent agreement**

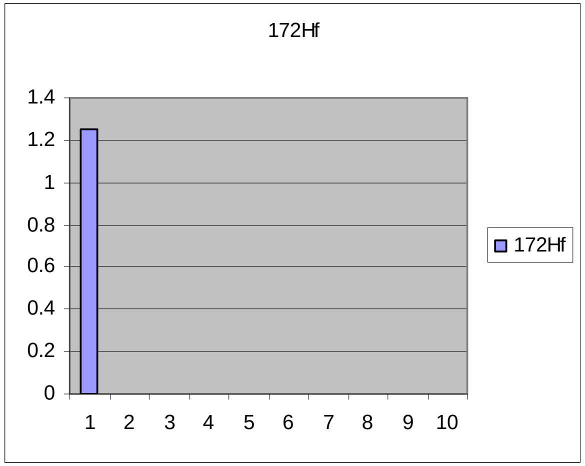

162Hf
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
162Hf

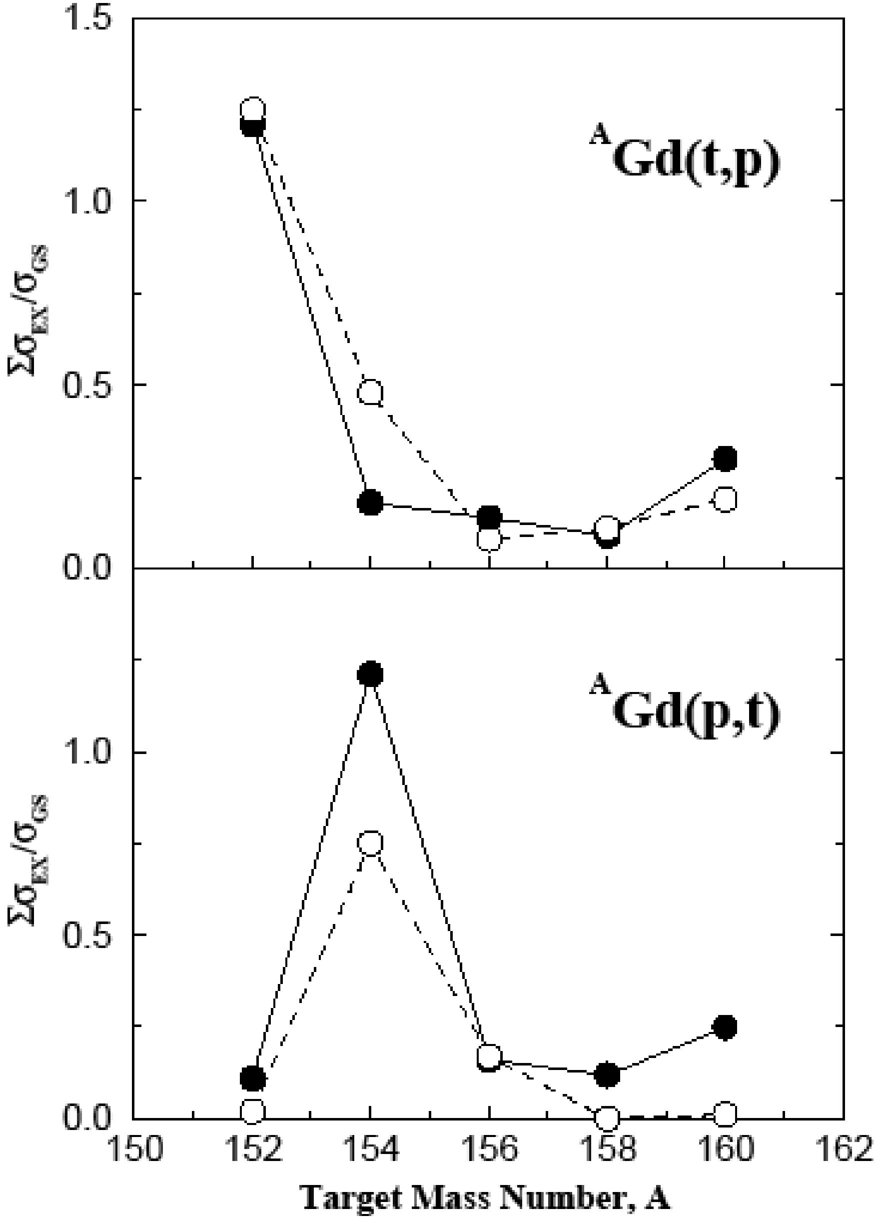

164Hf
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
164Hf


166Hf
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
166Hf


168Hf
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
168Hf


170Hf
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
170Hf

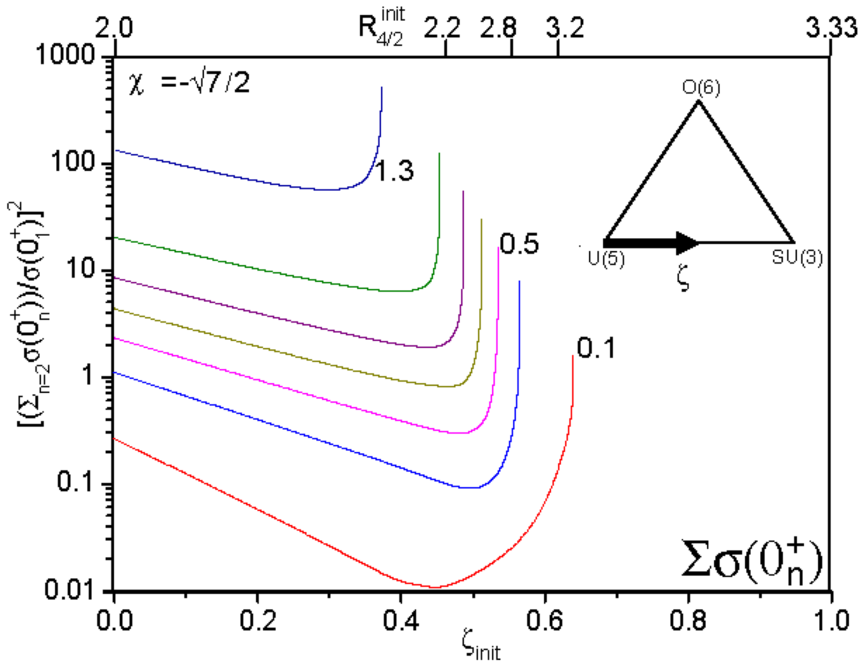

172Hf
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
172Hf



^{A}Gd(t,p)
^{A}Gd(p,t)
$\Sigma\sigma_{EX}/\sigma_{GS}$
$\Sigma\sigma_{EX}/\sigma_{GS}$
1.5
1.0
0.5
0.0
1.0
0.5
0.0
150
152
154
156
158
160
162
Target Mass Number, A

The model works well, so let’s look at its predictions for 2-nucleon transfer strengths throughout the triangle.

We will see that these predictions are radically different from the standard interpretation. They suggest an entirely different mechanism for the pair transfer, and give a single, general, unified, framework for both small and large cross sections.



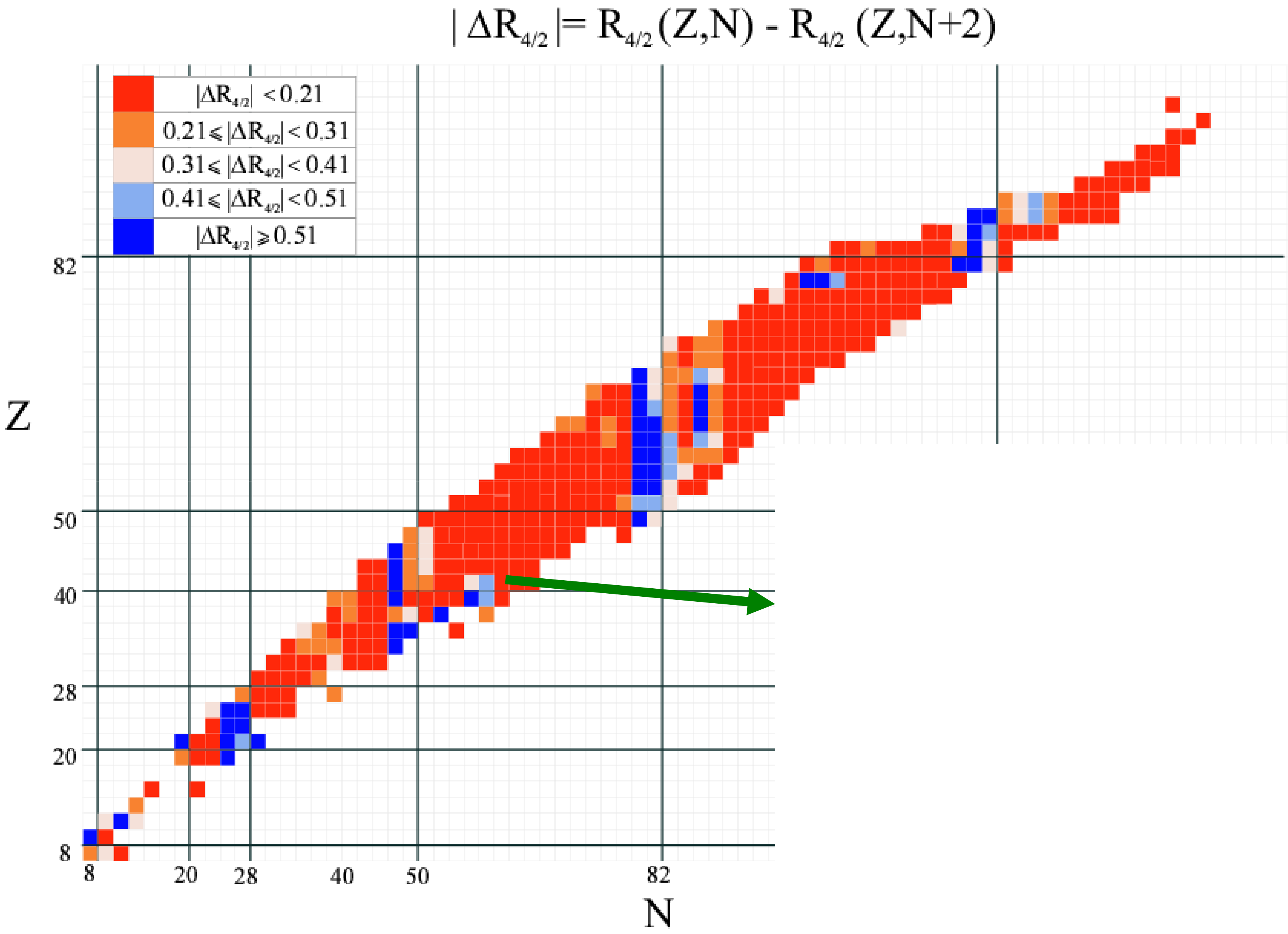
2.0
$R_{4/2}^{init}$
2.2
2.8
3.2
3.33
1000
100
10
1
0.1
0.01
χ =-√7/2
1.3
0.5
0.1
O(6)
U(5)
SU(3)
ζ
$\Sigma\sigma(0_n^+)$
$[(\Sigma_{n=2}\sigma(0_n^+))/\sigma(0_1^+)]^2$
0.0
0.2
0.4
0.6
0.8
1.0
ζ_{init}

Huh !!???

Thus, we find a completely unexpected result that leads to a new interpretation of these cross sections

- The cross sections are large in the transitional region but they are far larger in other cases. There is nothing special about the phase transition region.
- Rather, the cross sections depend only on the change in structure between initial and final nuclei ! This change can be “measured” by $\delta R_{4/2}$
- Can we test this? Can we find a case of large $\delta R_{4/2}$ that does **NOT** involve a phase transitional region.

Not easily but one case exists.

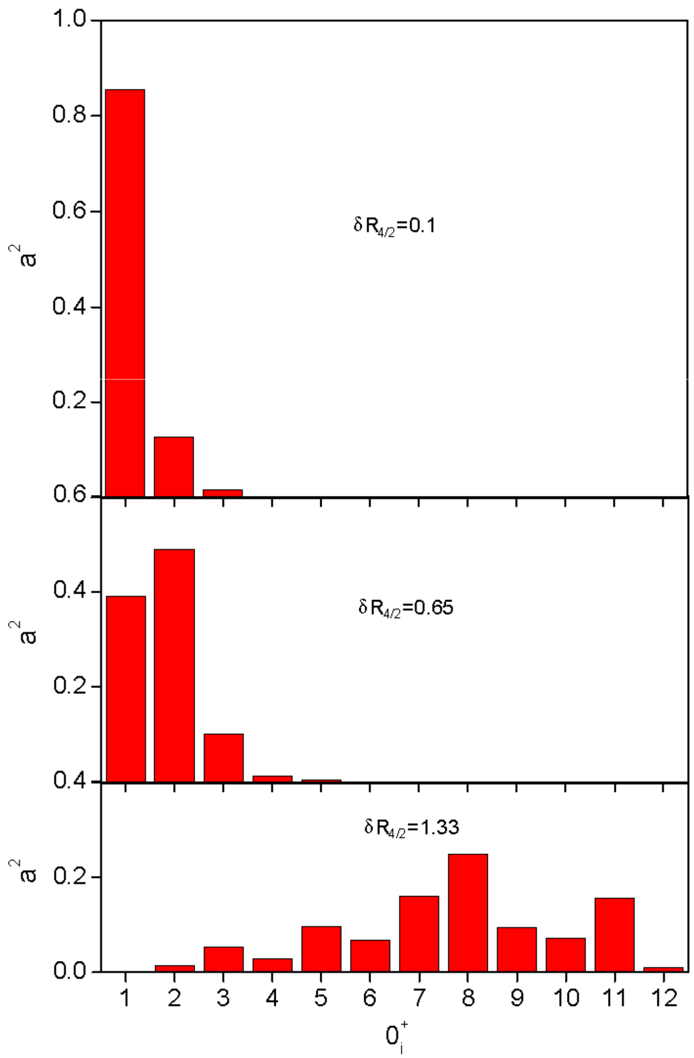


| ΔR4/2 |= R4/2 (Z,N) - R4/2 (Z,N+2)
|ΔR4/2| < 0.21
0.21 ≤ |ΔR4/2| < 0.31
0.31 ≤ |ΔR4/2| < 0.41
0.41 ≤ |ΔR4/2| < 0.51
|ΔR4/2| ≥ 0.51
Z
82
50
40
28
20
8
8
20
28
40
50
82
N

Physical explanation: The QQ term in the IBA Hamiltonian mixes the s and d boson basis states, spreading the strength. The further "apart" the two nuclei are, the greater the difference in the distributions ot s, d amplitudes, hence the greater the spreading of cross sectional strength to higher states.

Example: U(5) target: ground state has $(n_s, n_d) = (N,0)$. Therefore, only one amplitude contributes to cross section, that with $(n_s, n_d) = (N-1,0)$.

The figure shows the squared amplitudes for these components in each of the successive states for three different $\delta R_{4/2}$ values.



Two nucleon transfer cross sections and structural change in nuclei

Conclusions, Implications

- A single framework now accounts for both the (usual) small cross sections (since most adjacent nuclei have small $\delta R_{4/2}$ values), and for the large cross sections in regions of rapid change.

- The cross section distribution is a mixing effect but not of collective modes. Rather it is mixing at the shell model level (nucleon pairs coupled to spin 0) and therefore is general.

- Test in new nuclei by searching for large $\delta R_{4/2}$ values and doing 2-nucleon transfer in inverse kinematics.

Principal Collaborators

- Signatures of phase transitions:

  Dennis Bonatsos, Libby McCutchan

- Sensitivity of binding energies to structure:

  Burcu Cakirli, Klaus Blaum, Magda Kowalska

- Two – nucleon transfer cross sections:

  Rod Clark, Linus Bettermann, Ryan Winkler