Charge and mass distributions based on the three-dimensional TDHF calculations

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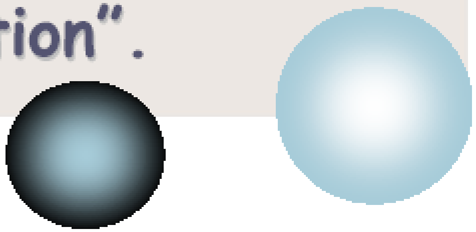
The concept: Charge equilibration

Charge equilibration is the homogenization of N/Z ratio during heavy-ion collisions.

Let us consider heavy-ion collisions of two different nuclei (more precisely, two nuclei with different N/Z ratio).

In some cases, N/Z ratio reaches the equilibrium during the collision, while it does not necessarily occurs. It depends on some conditions e.g. the bombarding energy, the impact parameter, and the sort of initial nuclei.

The equilibrium of N/Z ratio is called "**charge equilibrium**", and the process of reaching it during the collision is called "**charge equilibration**".



Charge equilibration

Standing problem more than 25 years …

French contribution is large (P. Bonche, C Simenel ...)

What is the essential mechanism of charge equilibration ?

= How can we understand this rapid process taking ~ 10^{-22} sec

We would like to clarify (bombarding) energy-dependence of charge equilibration ?

Does charge equilibration take place equivalently in reaction involving heavy nuclei ?

3-dimensional TDHF calculations

Systematic 3-dimensional TDHF calculations with respect to **"bombarding energy"** and **"impact parameter"** is performed for the following reactions:

Each 1MeV/A

Each 2.5fm

Ideal to be studied by time-dependent calculations

TDHF calculations with SLy4 interaction

^{208}Pb + ^{238}U

^{208}Pb + ^{132}Xe

^{208}Pb + ^{132}Sn

^{208}Pb + ^{40}Ca

^{208}Pb + ^{24}Mg

^{208}Pb + ^{24}O

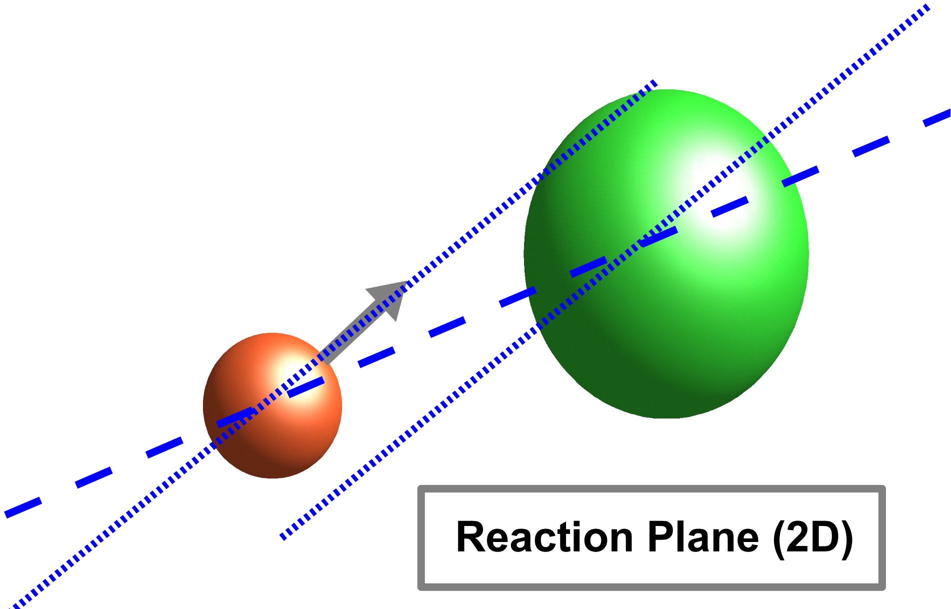
^{208}Pb + ^{4}He

----

^{132}Xe + ^{132}Sn, ^{24}O + ^{24}Mg

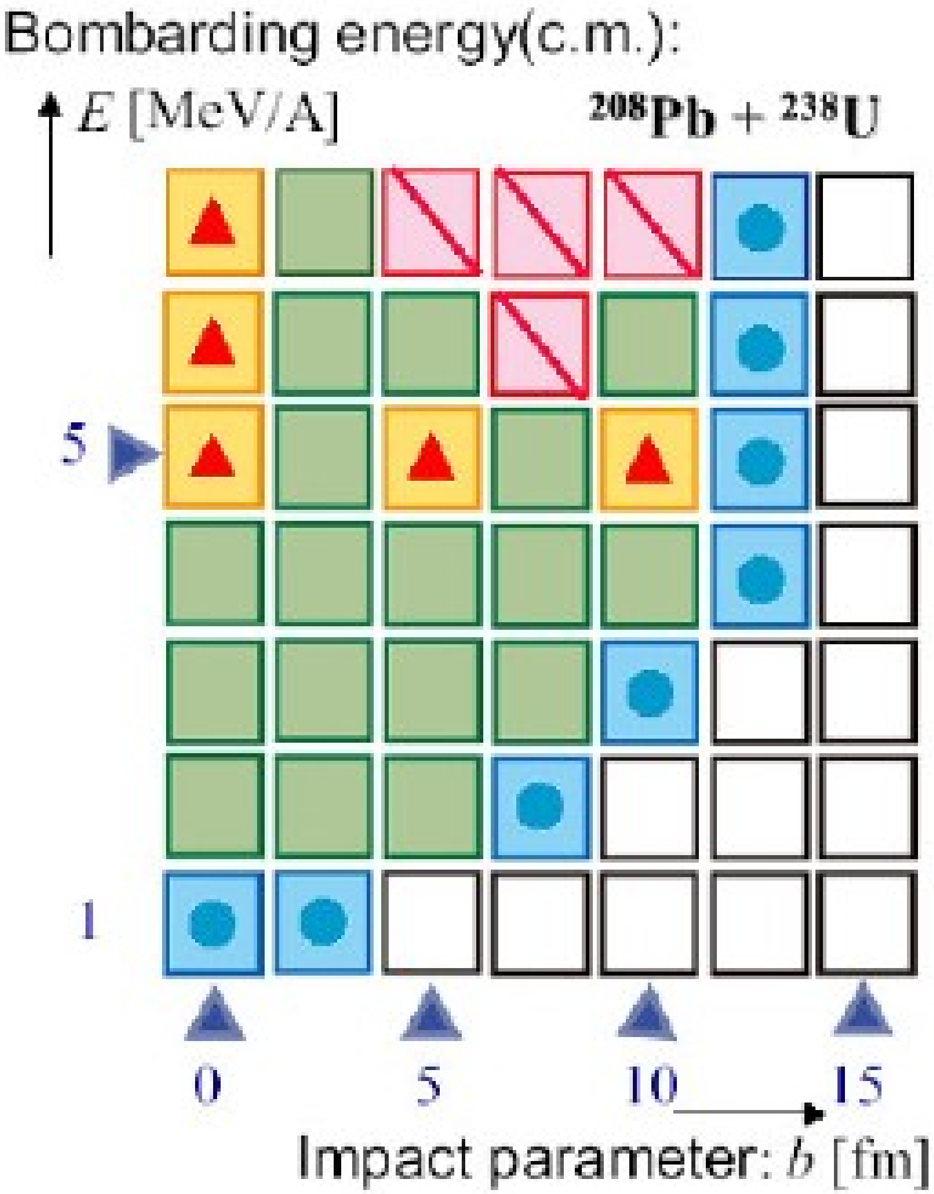
This is a large-scale calculation using ~ 30 CPU with 50GB memory (Cluster-Machine+others)

As a result, 1 ~ 3 months calculation time is necessary for each collision.



Systematic TDHF calculations

Sly4



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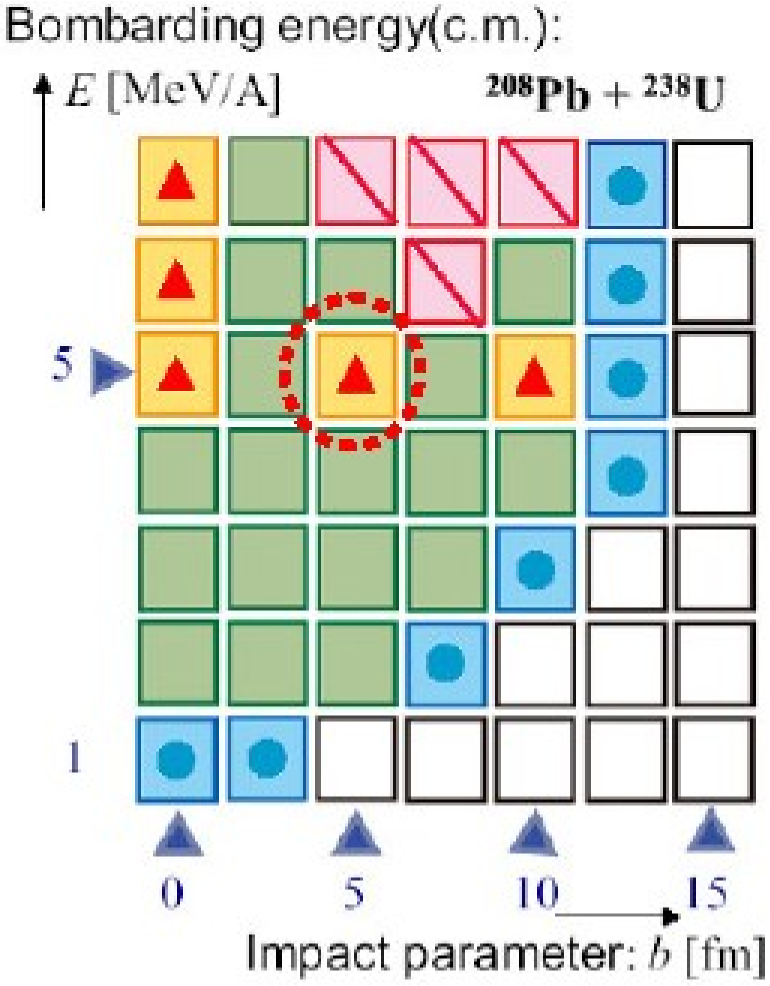
☐ no contact
● contact without any nucleon transfer
🟩 fragmentation into two pieces
▲ fragmentation into three pieces
⧄ fragmentation into more than three pieces

TDHF Calculations have been performed for b in steps of 2.5fm and Ec.m. in steps of 1 MeV/A.
Each box in the left figure corresponds to a single TDHF calculation, and color/markings distinguish the number of fragments.

As is shown in the figure, we have performed almost **50 different TDHF calculations** systematically.

Example (deep inelastic collision)

Sly4



Bombarding energy(c.m.):

E [MeV/A]

^{208}Pb + ^{238}U

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Impact parameter: b [fm]

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- ● contact without any nucleon transfer
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- ▲ fragmentation into three pieces
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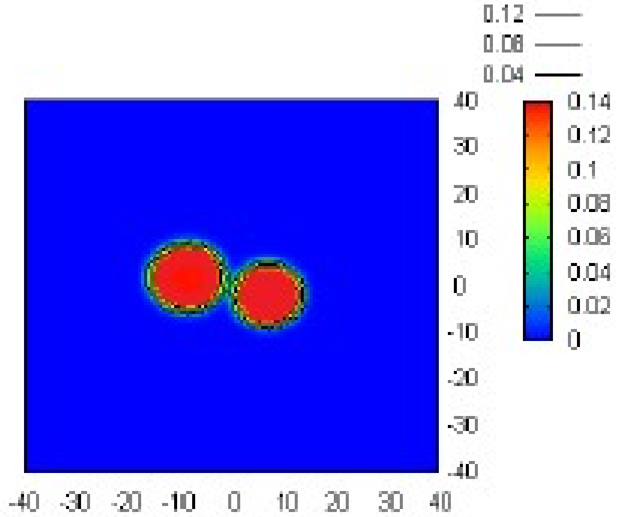
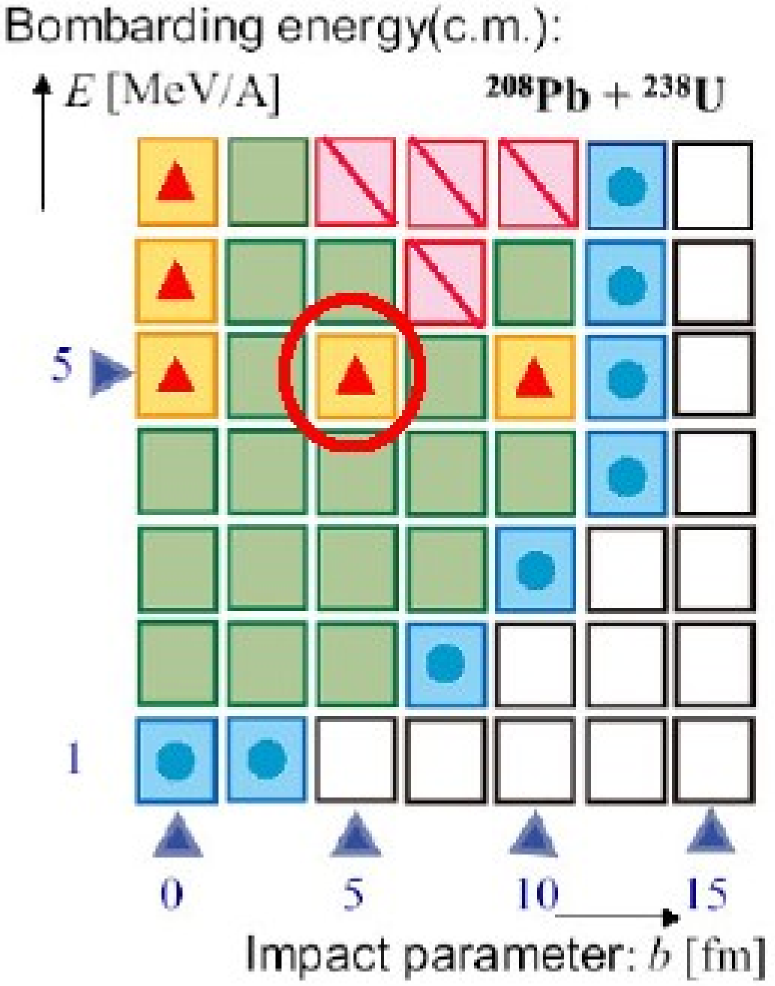


Example (deep inelastic collision)

Sly4



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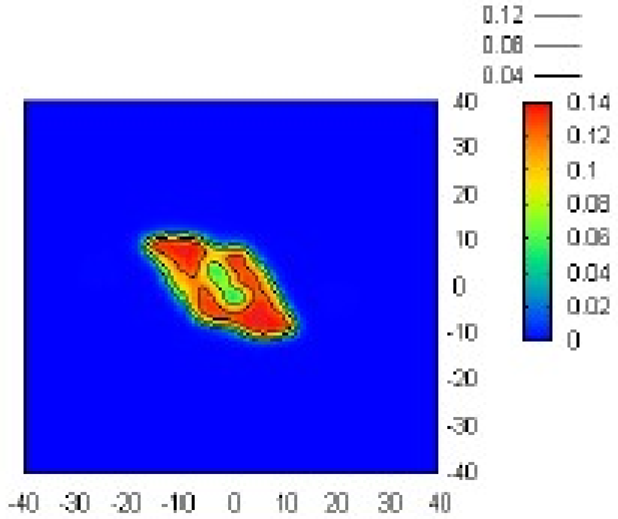
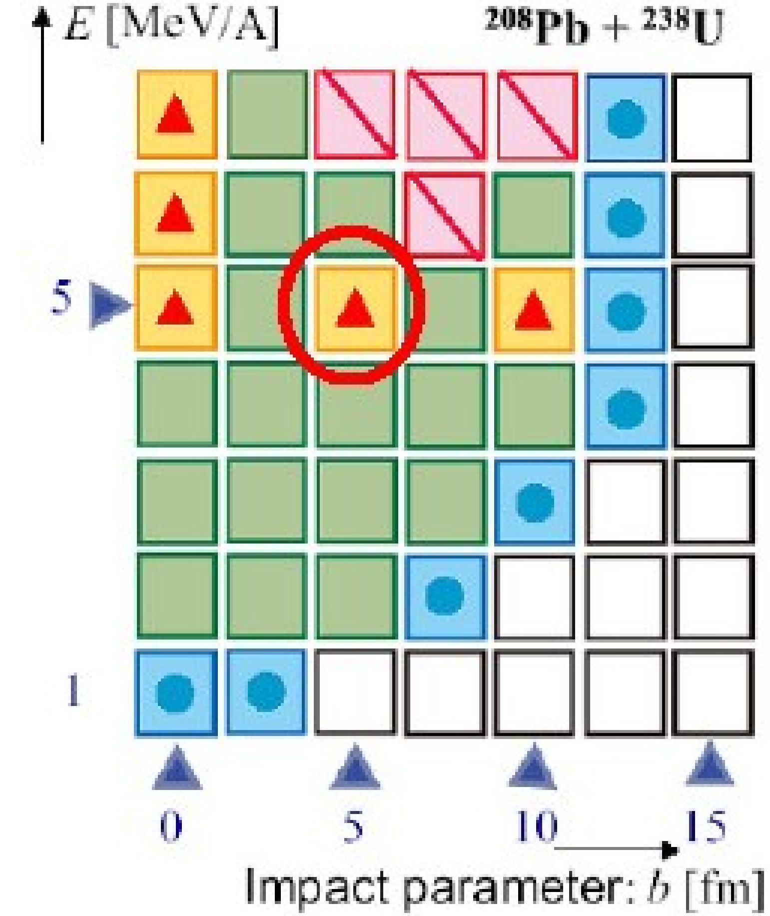


Example (deep inelastic collision)

Sly4

Bombarding energy(c.m.):



E [MeV/A]

$^{208}\mathrm{Pb} + {}^{238}\mathrm{U}$

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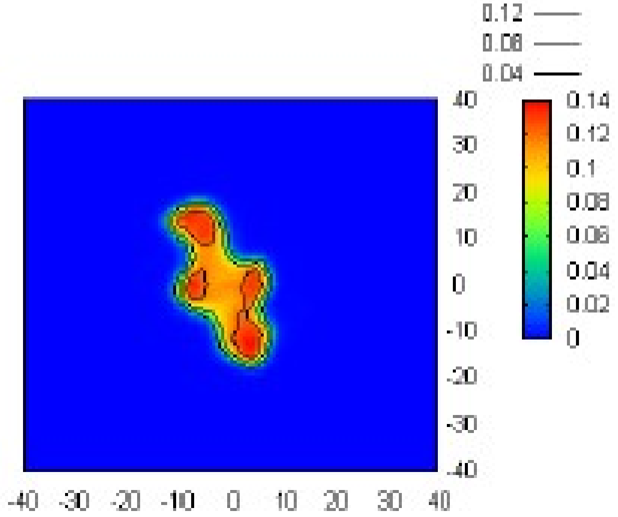
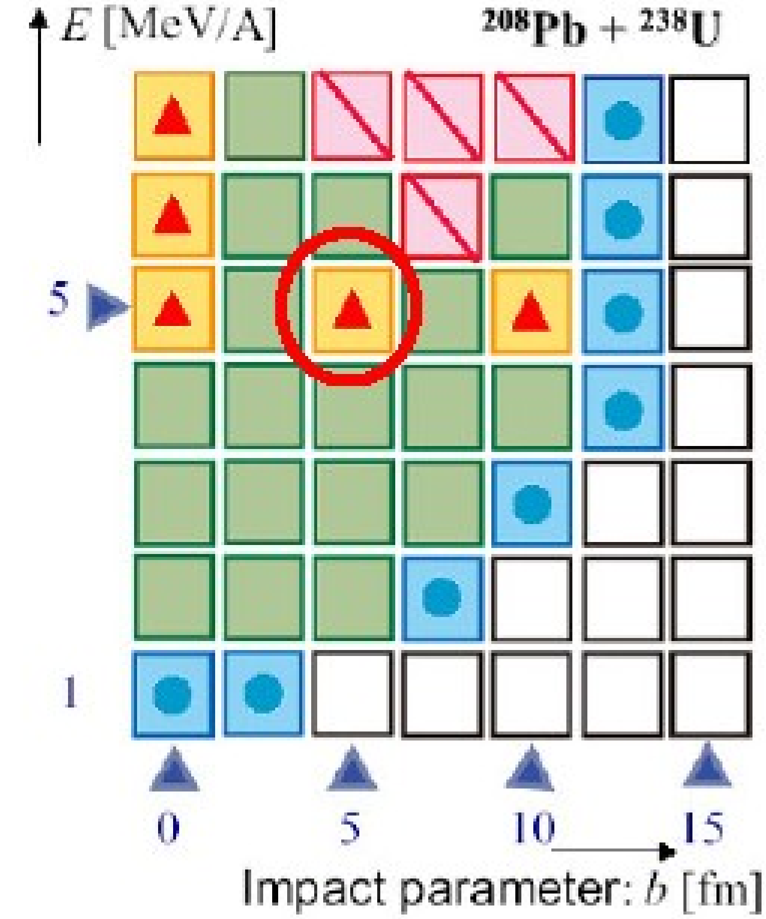


Example (deep inelastic collision)

Sly4

Bombarding energy(c.m.):



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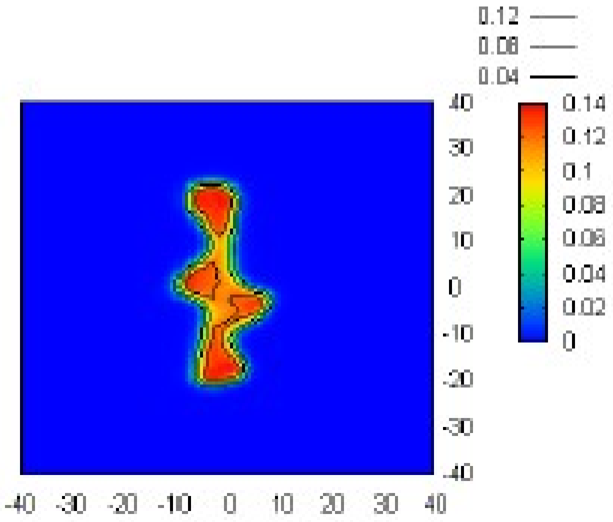
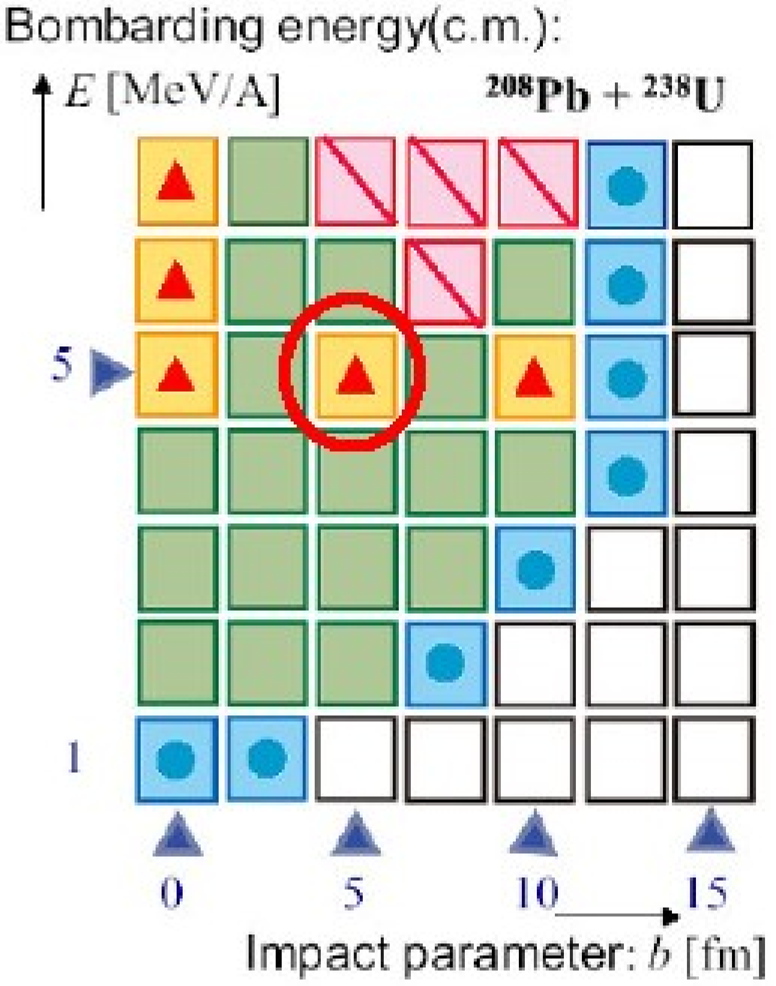


Example (deep inelastic collision)

Sly4



E [MeV/A]	0		5		10		15
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1	●	●	☐	☐	☐	☐	☐

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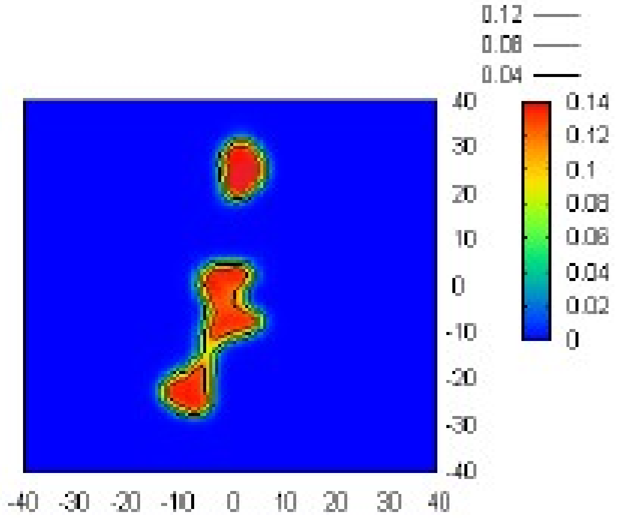


^{208}Pb + ^{238}U

Weight ~ πb_1^2, $\pi(b_2^2 - b_1^2)$, $\pi(b_3^2 - b_2^2)$, ...

Statistics is obtained by summing up different impact parameters.

Statistics of Final Products

Histograms of final products (impact parameters are summed up)

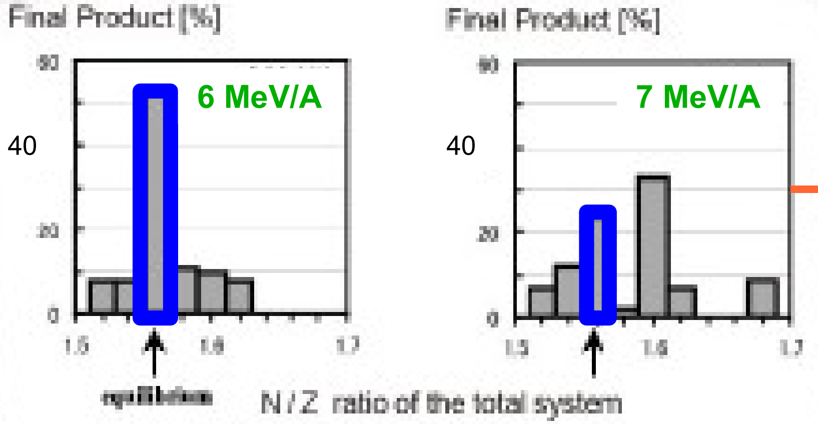


Fig.5-1 Distribution of N/Z ratio

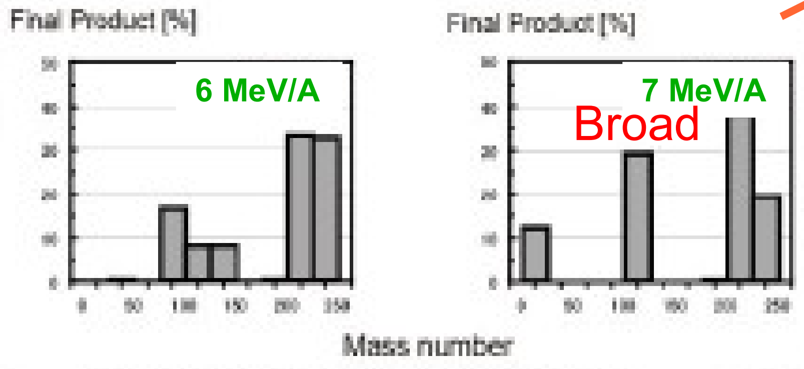


Fig.5-2 Distribution of Mass

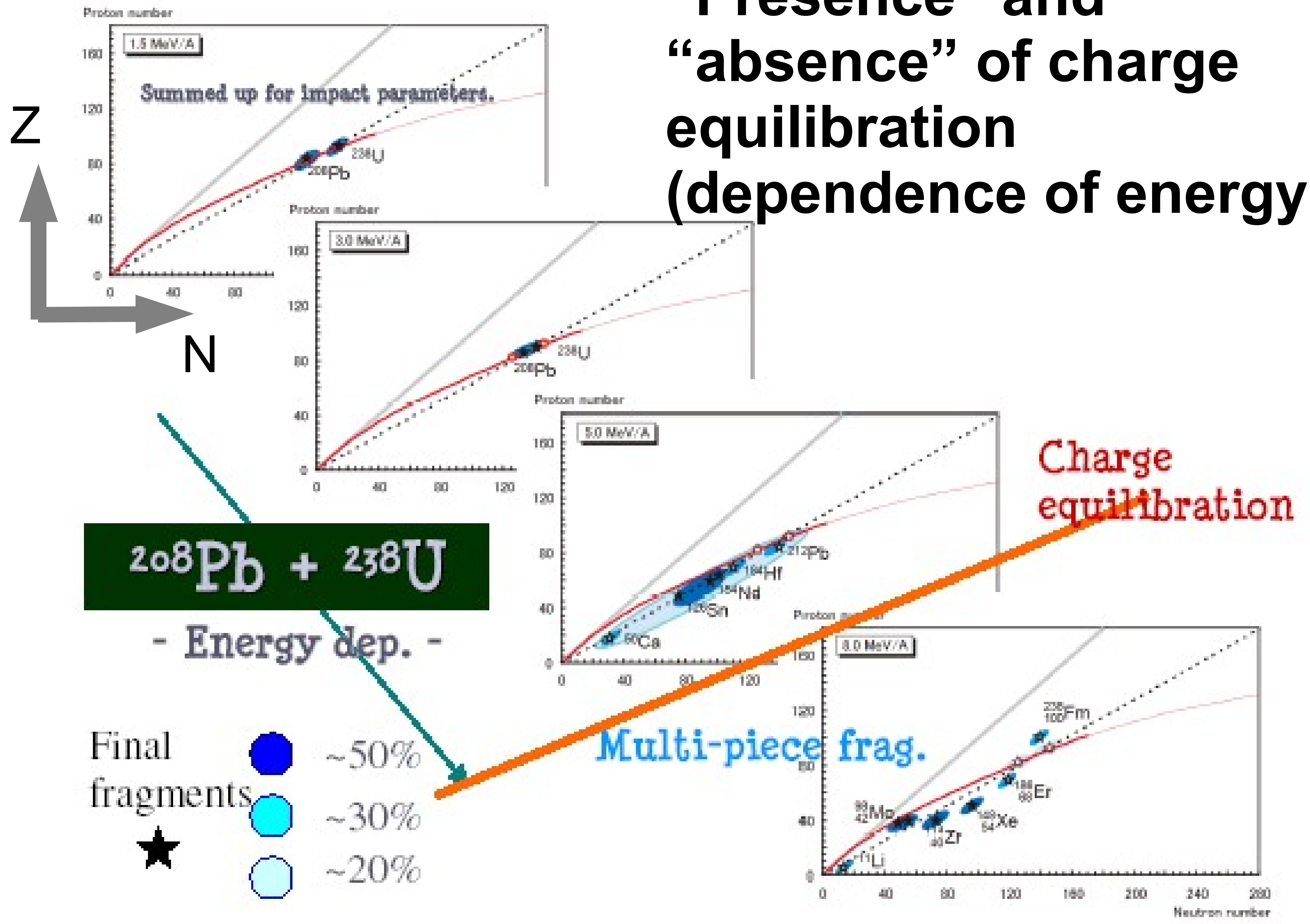
The TDHF results shown in figures 5-1 and 5-2 are the summed up in terms of the impact parameters.

From Fig.5-1, TDHF calculations show that 52% of the final products are equilibrated at 6MeV/A, but 23% at 7MeV/A. The shape of the N/Z distribution changes drastically.

From Fig.5-2, TDHF calculations show that the masses of final fragments are located around 110 and 230 for 6MeV/A, but 25, 100 and 230 for 7MeV/A. In the former case, the reaction is dominated by fragmentation into two or three pieces. In the latter case, fragmentation into four pieces appears, which produces lighter-mass fragments.

Pb:1.54 U:1.59

“Presence” and “absence” of charge equilibration (dependence of energy)



Charge equilibration

Multi-piece frag.

^{208}Pb + ^{238}U

- Energy dep. -

Final fragments ★

● ~50%

● ~30%

○ ~20%

Table 1. Summary of TDHF calculations

• Reaction	Upper energy-limit [Mev/A]
$^{208}Pb + {}^{238}U$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{132}Xe$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{132}Sn$	$6.0 < E_{CE} < 7.0$
$^{208}Pb + {}^{40}Ca$	$3.0 < E_{CE} < 4.0$
$^{208}Pb + {}^{24}Mg$	$2.0 < E_{CE} < 3.0$
$^{208}Pb + {}^{24}O$	$2.0 < E_{CE} < 3.0$
$^{208}Pb + {}^{16}O$	$1.0 < E_{CE} < 2.0$
$^{208}Pb + {}^{4}He$	$E_{CE} < 1.0$
$^{24}Mg + {}^{24}O$	$6.0 < E_{CE} < 7.0$

<u>Mass-dependence</u> can be found in this table.
How can we understand it ?

“Charge Equilibration” upper energy-limit Formula

By comparing **the Fermi velocity** with **the relative velocity** ...

If the touching time is large enough for a single nucleon with Fermi velocity to propagate throughout the two colliding nuclei, the charge equilibration can take place.

$$|\mathbf{v}_F| = \hbar|\mathbf{k}_F|/m = \hbar(3\pi^2\rho_{\min})^{1/3}/m, \qquad |\mathbf{v}_r| = \sqrt{\frac{2}{\mu}\left(E_{\mathrm{cm}} - \frac{1}{4\pi\epsilon}\frac{Z_1Z_2e^2}{r_0(A_1^{1/3}+A_2^{1/3})}\right)},$$

$$\rho_{\min} = \min_i\,(\rho_{ni}, \rho_{zi}) \quad = \min_i\left(\frac{N_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\epsilon)(1+\delta)}, \frac{Z_i\left(\frac{4\pi r_0}{3}A_i^{1/3}\right)^{-1}}{(1-3\epsilon)(1-\delta)}\right),$$

By solving $|v_r| = |v_F|$, we obtain the upper energy-limit (1).

Based on the Fermi-gas with the droplet model, we propose

$$E_{\mathrm{CE}} = \frac{\hbar^2\left(3\pi^2\rho_{min}\right)^{2/3}}{2m}\frac{A_1A_2}{A_1+A_2} + \frac{e^2}{4\pi\epsilon_0 r_0}\frac{Z_1Z_2}{A_1^{1/3}+A_2^{1/3}} \qquad (1)$$

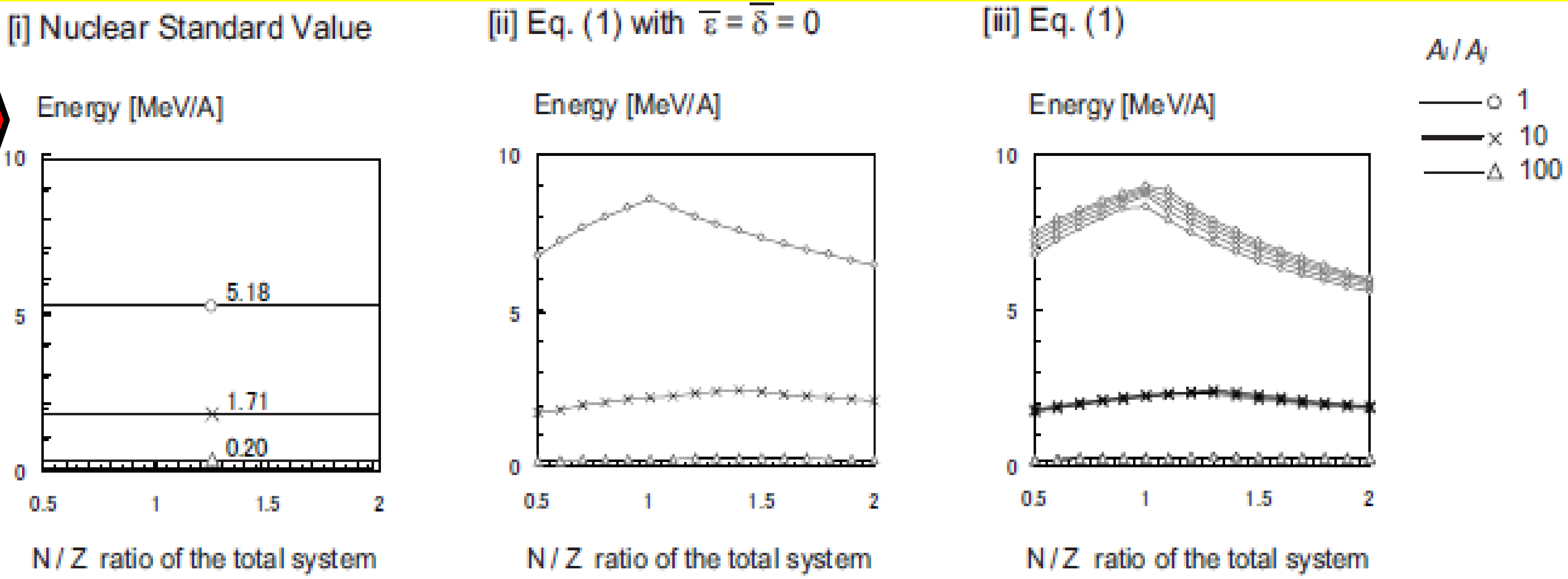
The derivation and the meaning of this formula should be referred to AIP conference proceedings 1098 (2009) 308

"Charge Equilibration" upper energy-limit Formula

$$E_{\mathrm{CE}} = \frac{\hbar^2 \left(3\pi^2 \rho_{min}\right)^{2/3}}{2m} \frac{A_1 A_2}{A_1 + A_2} + \frac{e^2}{4\pi\epsilon_0 r_0} \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}}$$

$$|\vec{v}_F| = \frac{\hbar |\vec{k}_F|}{m} = \frac{\hbar}{m}\left(3\pi^2 \rho_{\mathrm{min}}\right)^{1/3}$$

Graphical representation


[i] Nuclear Standard Value

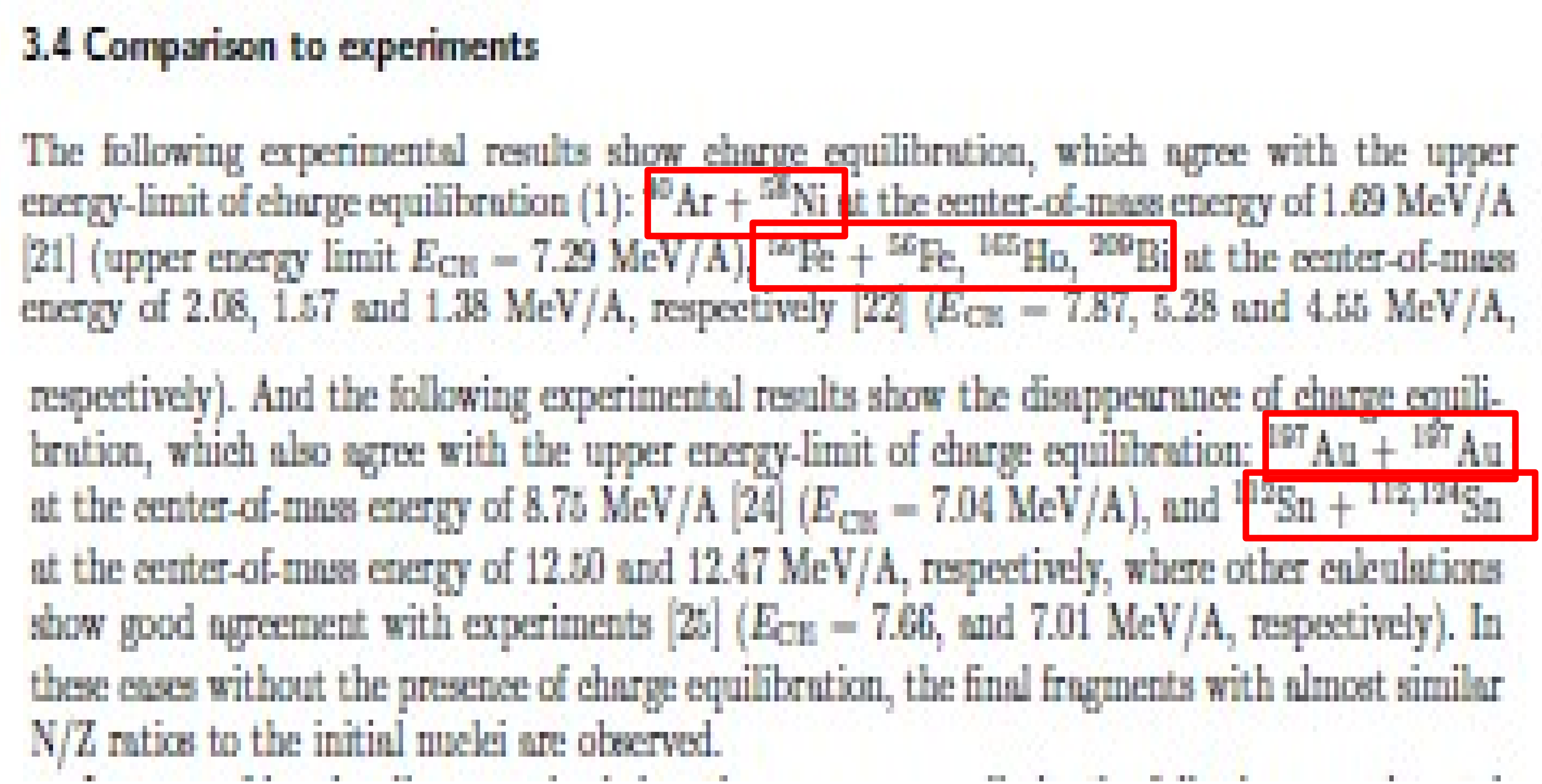
[ii] Eq. (1) with $\bar{\varepsilon} = \bar{\delta} = 0$

[iii] Eq. (1)

$$k_F = 1.36\ \mathrm{fm}^{-1}$$

Coulomb and symmetry energies are taken into account

Agreement with Experiments



3.4 Comparison to experiments

The following experimental results show charge equilibration, which agree with the upper energy-limit of charge equilibration (1): ^{40}Ar + ^{58}Ni at the center-of-mass energy of 1.69 MeV/A [21] (upper energy limit $E_{CE} = 7.29$ MeV/A), ^{56}Fe + ^{56}Fe, ^{165}Ho, ^{209}Bi at the center-of-mass energy of 2.08, 1.57 and 1.38 MeV/A, respectively [22] ($E_{CE} = 7.87$, 5.28 and 4.55 MeV/A, respectively). And the following experimental results show the disappearance of charge equilibration, which also agree with the upper energy-limit of charge equilibration: ^{197}Au + ^{197}Au at the center-of-mass energy of 8.75 MeV/A [24] ($E_{CE} = 7.04$ MeV/A), and ^{112}Sn + ^{112,124}Sn at the center-of-mass energy of 12.50 and 12.47 MeV/A, respectively, where other calculations show good agreement with experiments [25] ($E_{CE} = 7.66$, and 7.01 MeV/A, respectively). In these cases without the presence of charge equilibration, the final fragments with almost similar N/Z ratios to the initial nuclei are observed.

For any case, we have an nice agreement with TDHF and Experimental results with respect C. E. Upper Limit.

Summary of charge equilibration

The Fermi energy is essential for determining **the upper-energy limit(presence or absence)**, although the symmetry is a secondary factor for this limit.
> we notice that the symmetry energy still be (only one) driving force for the charge equilibration

⇩

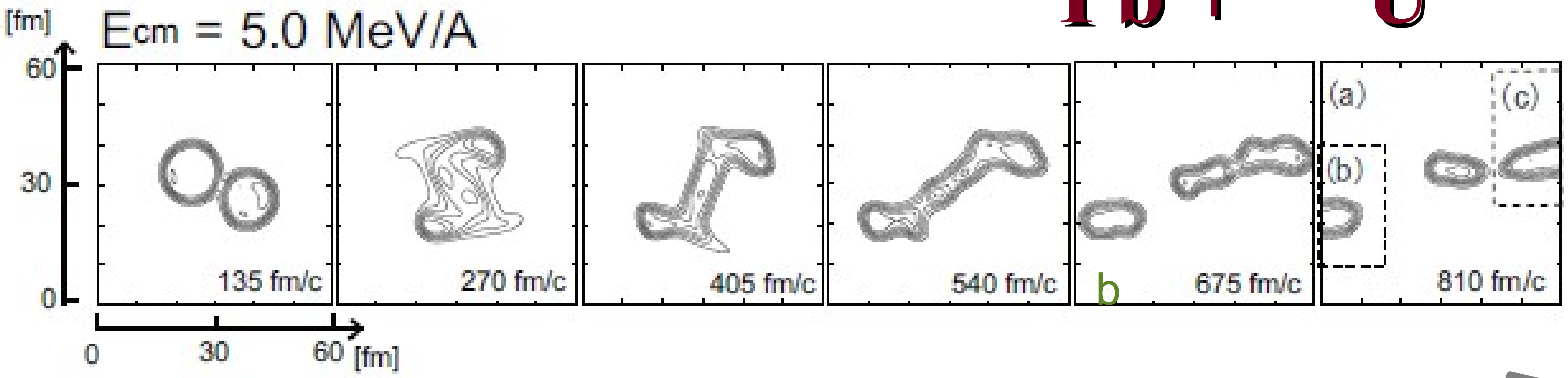
Mechanism ...

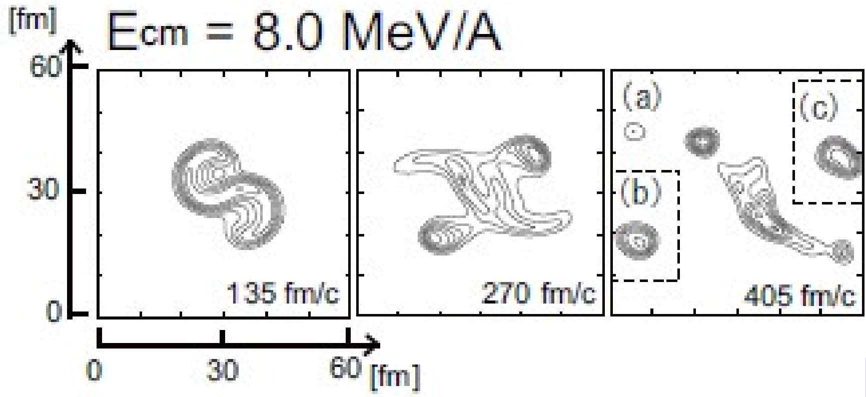
★ The charge equilibration is a rapid process ~ 10^{-22} sec >propagating with almost the Fermi velocity (Charge eq. is achieved by the nucleons with the Fermi velocity)

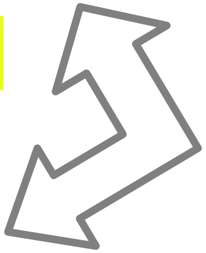
★ The charge equilibration similarly occurs in the collisions involving very heavy nuclei, although there exists a large Coulomb repulsion leading to the localization of charges. **The mass distribution is totally different between heavy and light reactions (as is discussed in the followings).**

Application of Charge Eq. formula (exotic production)

^{208}Pb + ^{238}U



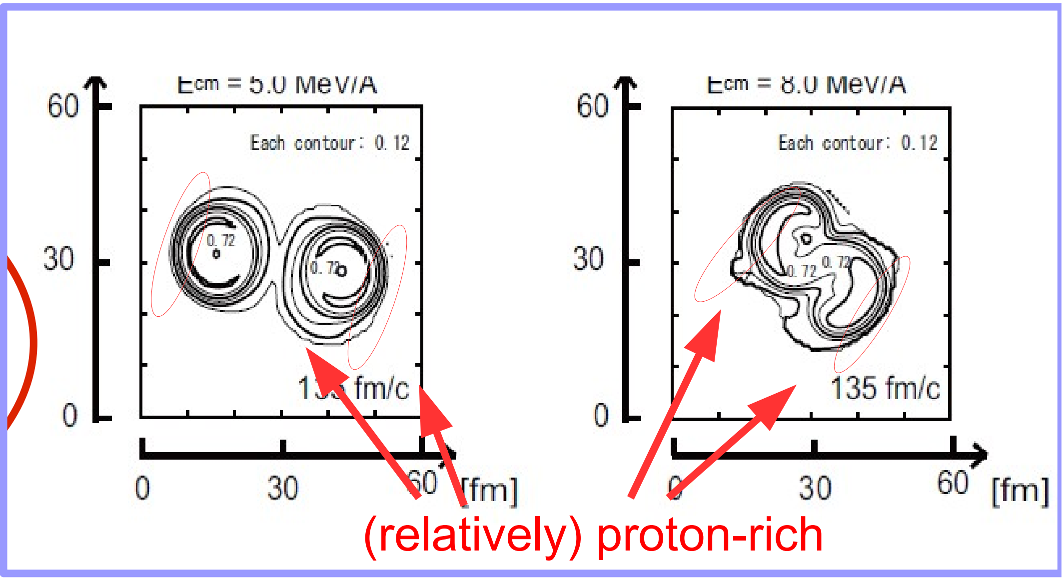




Charge Eq. true

Charge Eq. false

Z/N ratio at the contact time



(relatively) proton-rich

	(a)	(b)	(c
E_{cm} = 5.0 MeV/A	1.54	1.56	1.58
E_{cm} = 8.0 MeV/A	1.64	1.42	1.47

n-rich

p-rich

208Pb: 1.54
238U: 1.59

Summary for Exotic Nuclear Production

More exotic nuclei (both neutron and proton rich nuclei) are drastically produced above the upper energy limit.

Charge equilibrium is not achieved in the higher energies

The mass of final fragments inevitably become small.

It is necessary to understand the relation between mass and charge distributions.

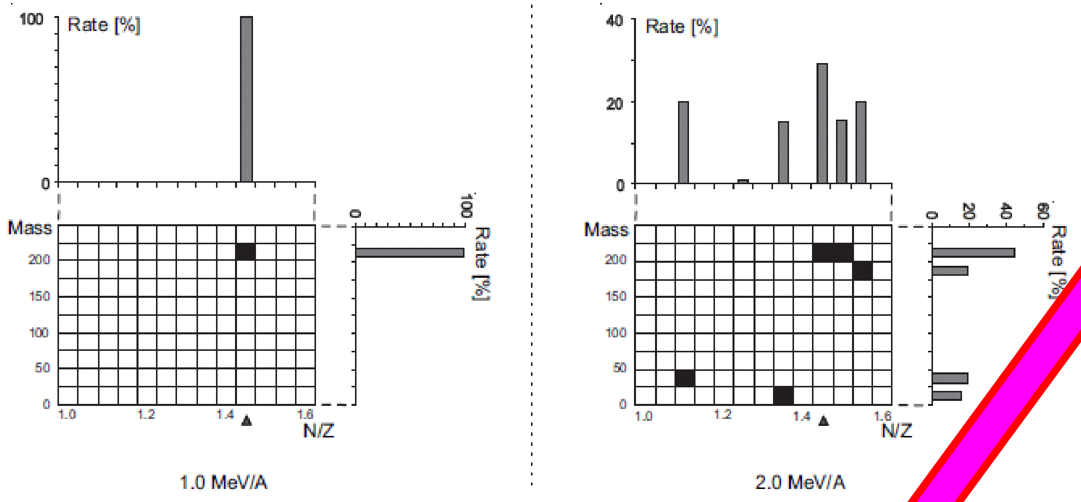
	(a)	(b)	(c)
$E_{cm} = 5.0$ MeV/A	1.54	1.56	1.58
$E_{cm} = 8.0$ MeV/A	1.64	1.42	1.47

n-rich

p-rich

208Pb: 1.54
238U: 1.59

Mass and charge distributions



1.0 MeV/A

2.0 MeV/A

Fig. 7. Histogram of final fragments (weighted by the geometric cross section) in[illegible]ral collisions between ^{208}Pb and ^{16}O, where the horizontal and vertical axes mean N/Z ratio an[illegible] mass number, respectively. N/Z ratio is incremented by 0.05 and the mass number by 25. Bl[illegible]or White square shows the existence (more than 5 %) or non-existence of final products. Triang[illegible]signate the charge equilibrium. Cases of E_{cm} = 1.0 and 2.0 MeV/A are shown.

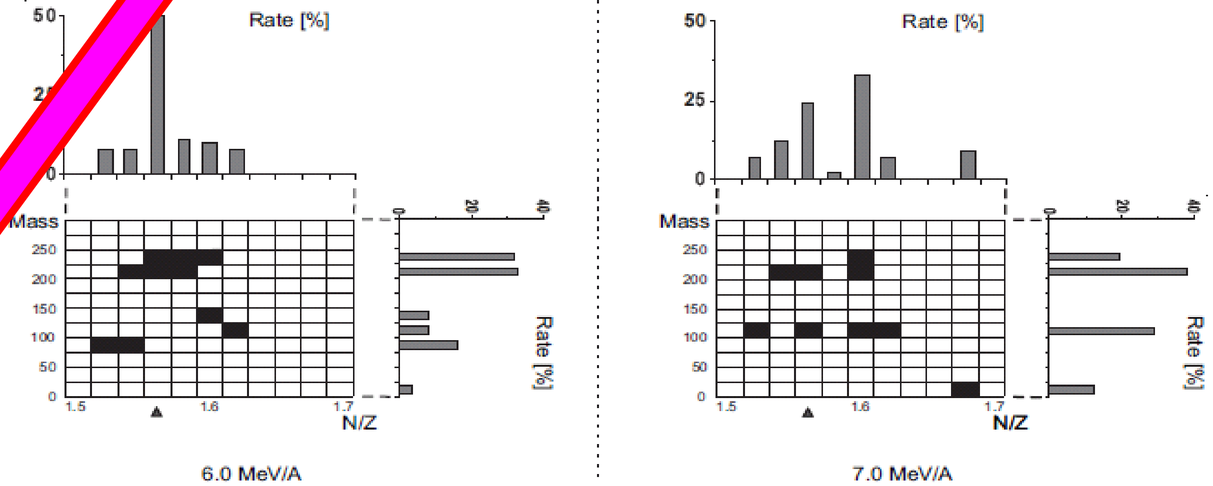
Statistics is obtained by summing up different impact parameters.

Heavier case

^{208}Pb + ^{238}U

Lighter case

^{208}Pb + ^{16}O



6.0 MeV/A

7.0 MeV/A

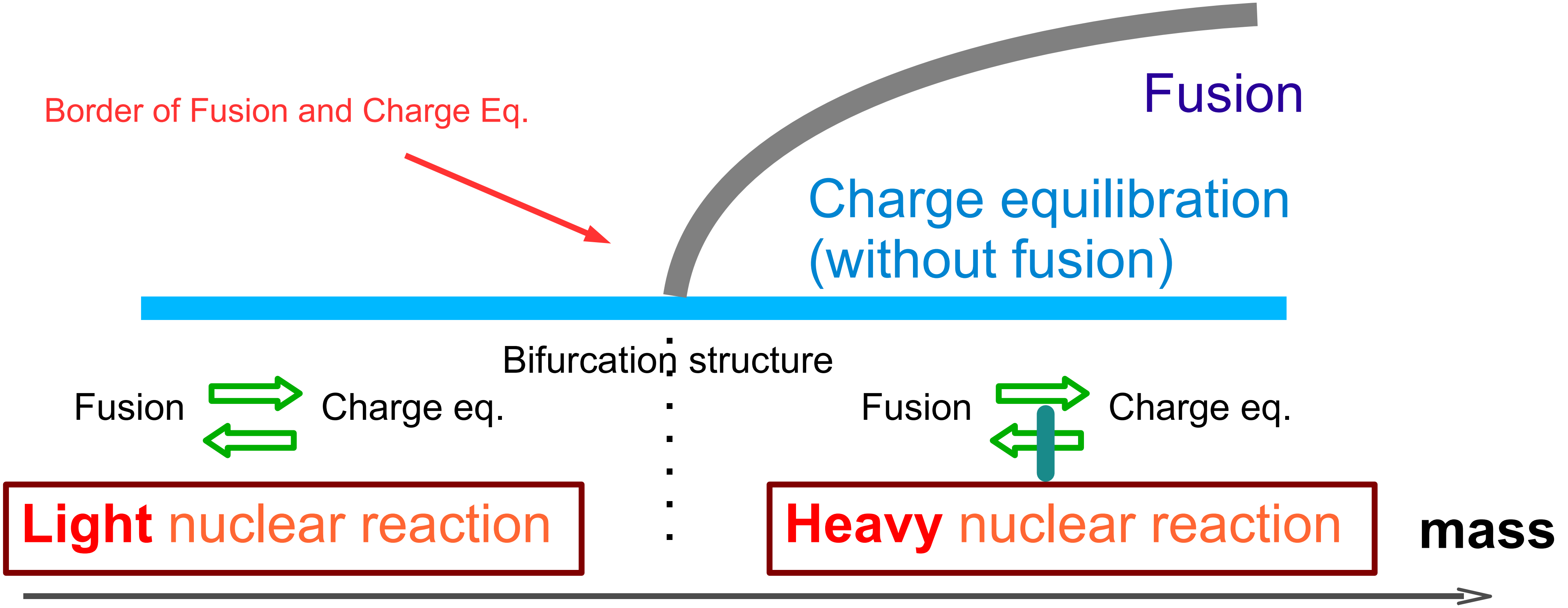
Fig. 8. Histogram of final fragments (weighted by the reaction cross section of classic mechanics) in central collisions between ^{208}Pb and ^{238}U, where the horizontal and vertical axes mean N/Z ratio and the mass number, respectively. N/Z ratio is incremented by 0.05 and the mass number by 25. Black or White square shows the existence (more than 5 %) or non-existence of final products. Triangles designate the charge equilibrium. Cases of E_{cm} = 6.0 and 7.0 MeV/A are shown.

Mass and charge distributions

To see the detail of mass distribution,

- We have verified the discrepancy of “fusion” and “charge equilibration” based on three-dimensional TDHF.

We point out the importance of the formation of the merged system at the intermediate stage



Border of Fusion and Charge Eq.

Fusion

Charge equilibration
(without fusion)

Bifurcation structure

Fusion ⇄ Charge eq.

Fusion ⇄ Charge eq.

Light nuclear reaction

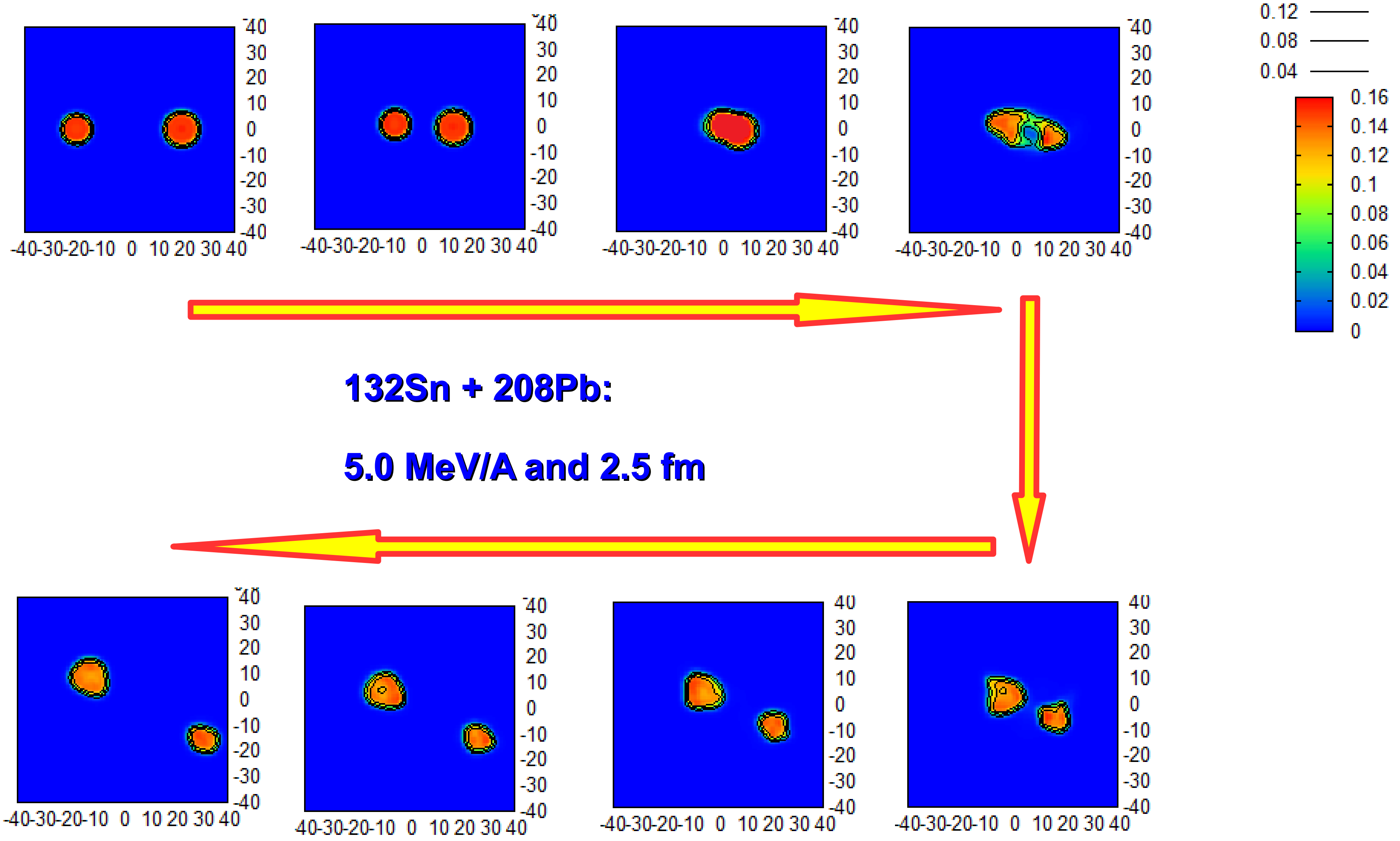
Heavy nuclear reaction

mass

Reactions, which play outstanding role in giving the discrepancy (~ Deep inelastic collisions)

132Sn + 208Pb --- Coulomb barrier: 1.3 MeV/A

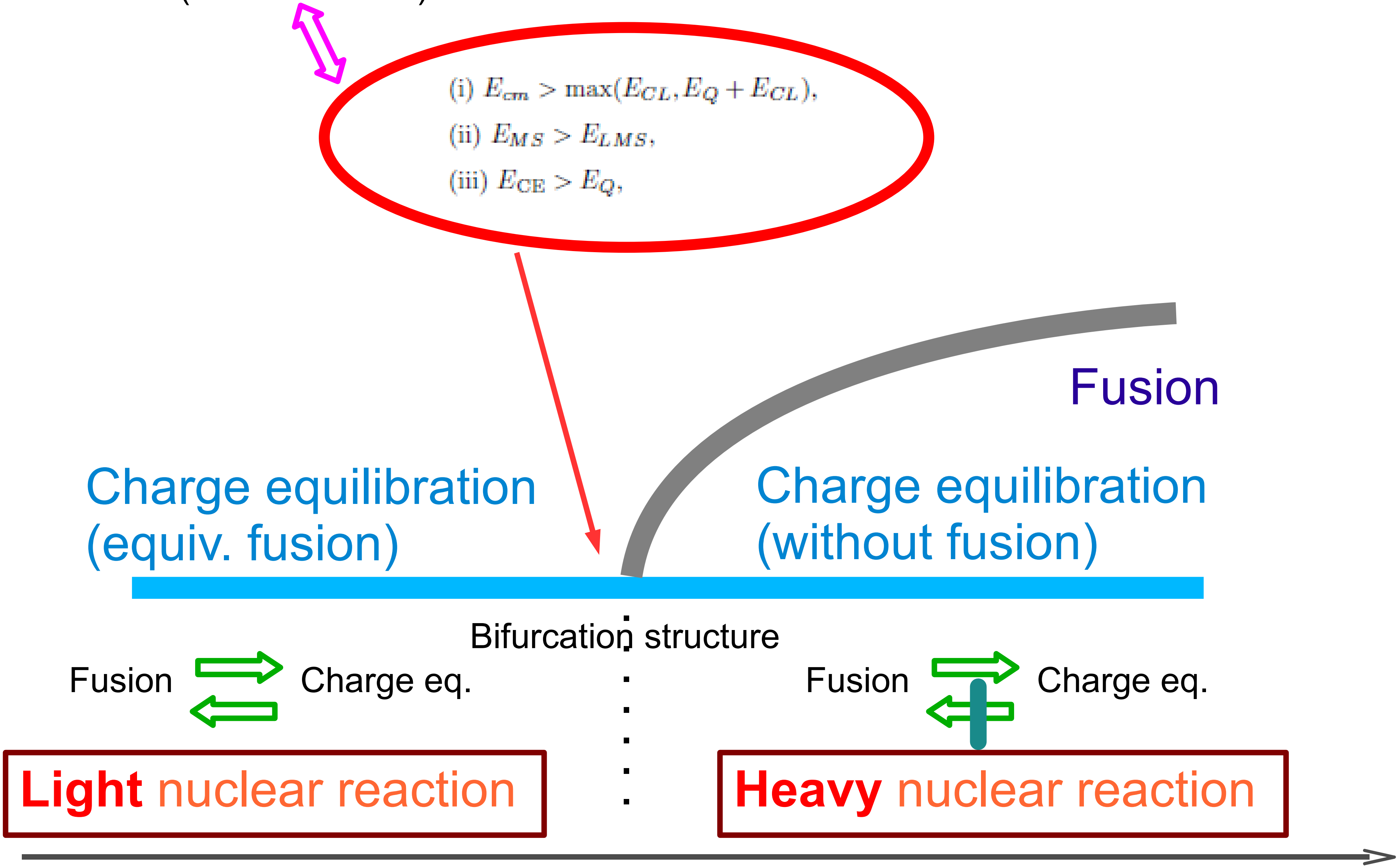
Upper energy-limit of charge equilibration: 6.4 MeV/A



132Sn + 208Pb:

5.0 MeV/A and 2.5 fm

The condition for the discrepancy between "Fusion" and "Charge Equilibration" is proposed and it is verified by TDHF3d and experiments (Iw et. al. EPJA)



(i) $E_{\mathrm{cm}} > \max(E_{CL}, E_Q + E_{CL})$,

(ii) $E_{MS} > E_{LMS}$,

(iii) $E_{\mathrm{CE}} > E_Q$,

Fusion

Charge equilibration (equiv. fusion)

Charge equilibration (without fusion)

Bifurcation structure

Fusion ⇄ Charge eq.

Fusion ⇄ Charge eq.

Light nuclear reaction

Heavy nuclear reaction

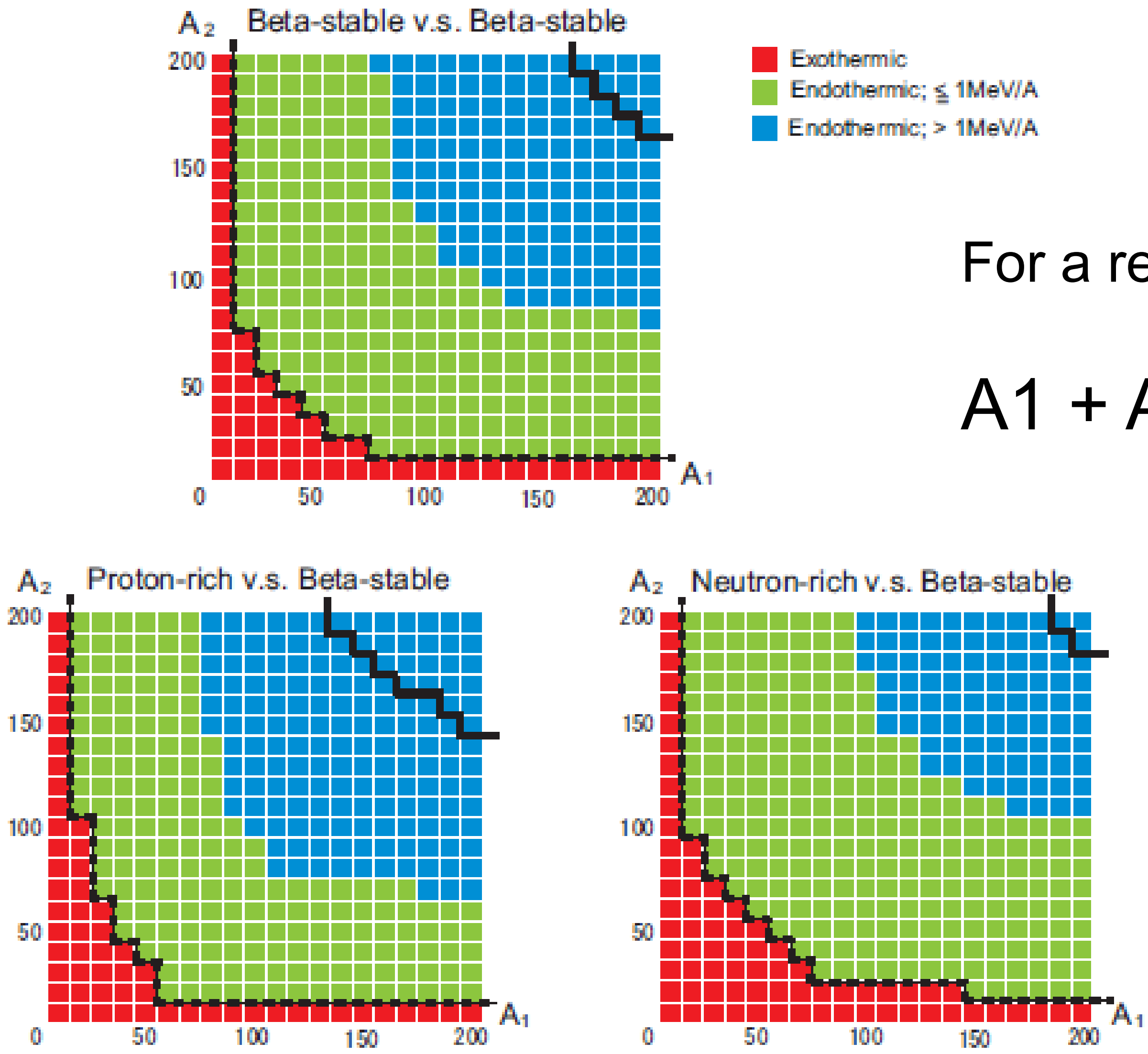


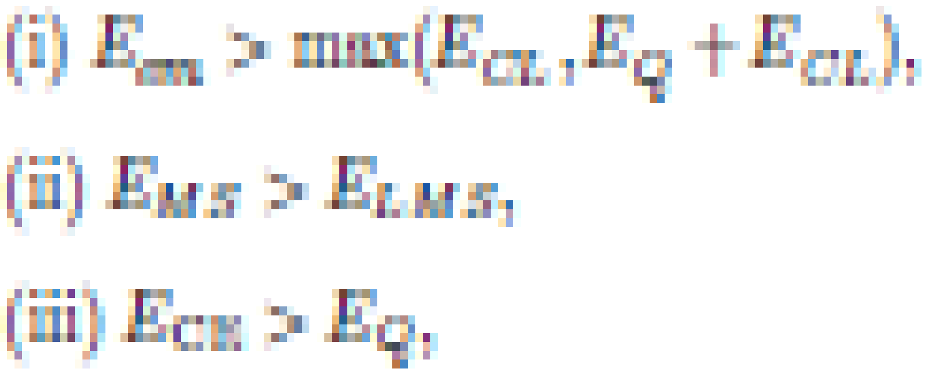
Fig. 1. (Color online) E_Q-values are shown by the color of each box, where the horizontal axes always mean the stable-nuclei. The borders of exothermic and endothermic reactions are shown by dotted lines. The merged system cannot be achieved on the upper right side of the solid lines. The neutron-rich and proton-rich nuclei are defined by the nuclei with 20 % larger and smaller N/Z ratio compared to the β-stable nuclei, respectively.

For a reaction:

A1 + A2 →

from Iw et. al. EPJA

Condition for the Discrepancy between Fusion and Charge Equilibration (Detail is omitted)

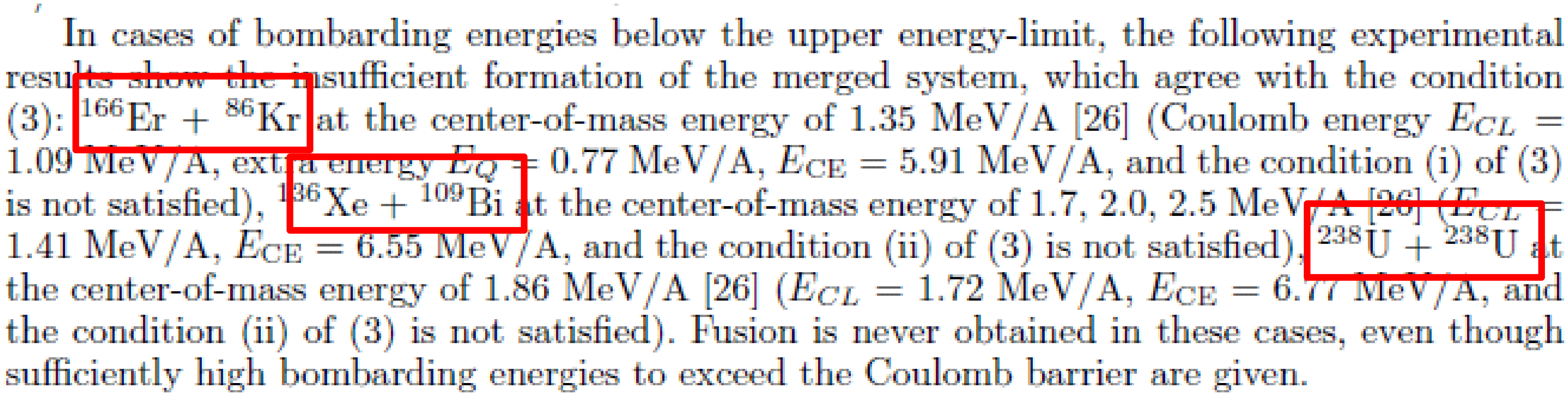


$$\text{(i) } E_{cm} > \max(E_{CL}, E_Q + E_{CL}),$$

$$\text{(ii) } E_{MS} > E_{LMS},$$

$$\text{(iii) } E_{CE} > E_Q,$$

Three simple conditions about **bombarding and binding energies**, and **charge equilibration upper-limit.**.
It also agrees well with experiments.



In cases of bombarding energies below the upper energy-limit, the following experimental results show the insufficient formation of the merged system, which agree with the condition (3): ^{166}Er + ^{86}Kr at the center-of-mass energy of 1.35 MeV/A [26] (Coulomb energy E_{CL} = 1.09 MeV/A, extra energy E_Q = 0.77 MeV/A, E_{CE} = 5.91 MeV/A, and the condition (i) of (3) is not satisfied), ^{136}Xe + ^{109}Bi at the center-of-mass energy of 1.7, 2.0, 2.5 MeV/A [26] (E_{CL} = 1.41 MeV/A, E_{CE} = 6.55 MeV/A, and the condition (ii) of (3) is not satisfied), ^{238}U + ^{238}U at the center-of-mass energy of 1.86 MeV/A [26] (E_{CL} = 1.72 MeV/A, E_{CE} = 6.77 MeV/A, and the condition (ii) of (3) is not satisfied). Fusion is never obtained in these cases, even though sufficiently high bombarding energies to exceed the Coulomb barrier are given.

Proposed condition also means that ...

^{208}Pb + ^{4}He

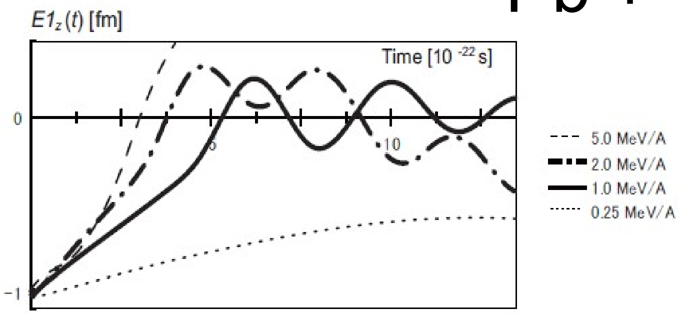


Fig. 4. Time evolution of dipole moment in central collisions between ^{208}Pb and ^{16}O, where the cases of E_{cm} = 0.25, 1.0, 2.0, and 5.0 MeV/A are shown (from lower to upper). The values are normalized by the initial value $E1_z(0)$ (the same for Figs. 5 and 6).

^{208}Pb + ^{40}Ca

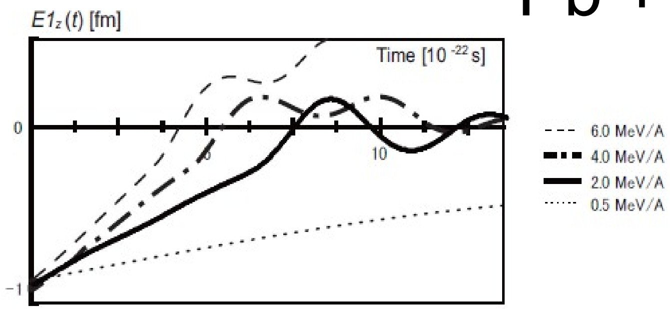


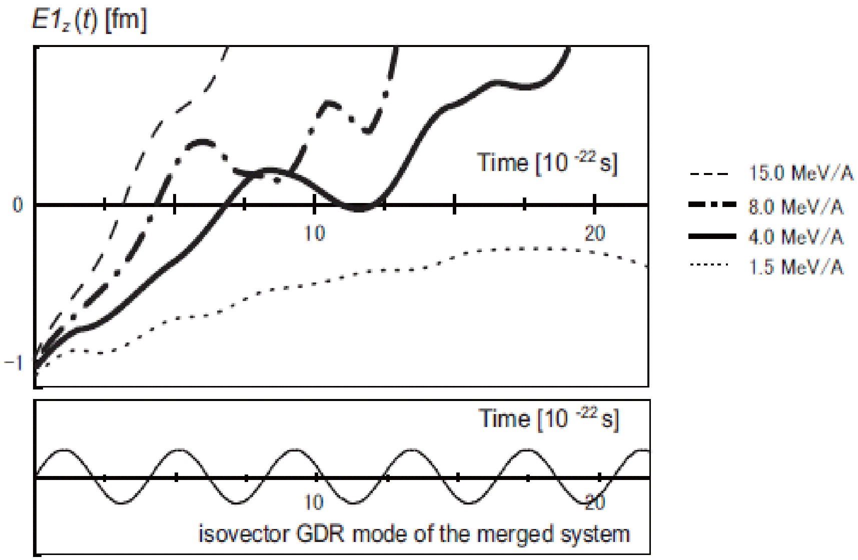
Fig. 5. Time evolution of dipole moment in central collisions between ^{208}Pb and ^{40}Ca, where the cases of E_{cm} = 0.5, 2.0, 4.0, and 6.0 MeV/A are shown (from lower to upper).

The condition is **satisfied**

We have shown charge equilibration is empowered by the mean-field effect, while it is sometimes independent of the iv-GDR .

from Iw et. al. EPJA

The condition **is not satisfied**



^{208}Pb + ^{238}U

Fig. 6. (Upper) Time evolution of dipole moment in central collisions between ^{208}Pb and ^{238}U, where the cases of E_{cm} = 1.5, 4.0, 8.0, and 15.0 MeV/A are shown. (Lower) For reference, oscillation with isovector-GDR mode of the merged system ^{446}174 is shown in a simplified manner, where only the frequency of GDR is reproduced as the sine function.

Summary

(verified by TDHF3d and Experiments)

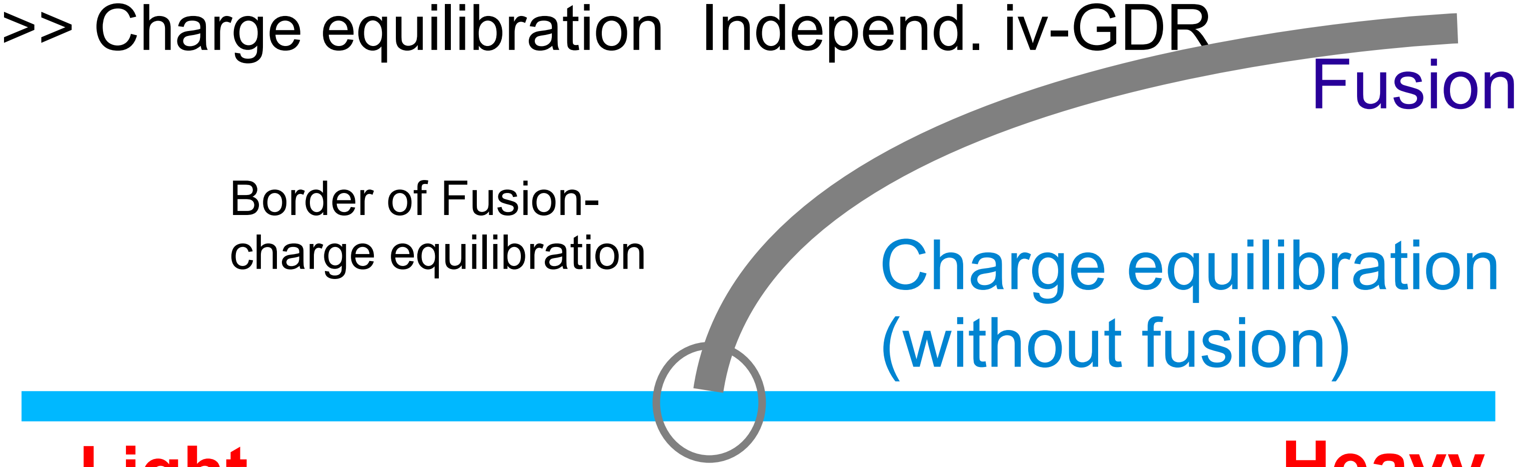
1) Charge equilibration upper energy-limit formula

>>> **Fermi energy** is the first essential factor. It corresponds nicely to the rapid process taking only 10^{-22} sec (mechanism). It can be the basis for **exotic nuclear synthesis**.

2) Discrepancy between fusion and charge equilibration

>>> The condition is proposed. The most important factor is **the formation of the merged composite nuclei** at the early stage of heavy-ion collisions. The condition for that boundary is proposed.

>>> Charge equilibration Independ. iv-GDR



Fusion

Border of Fusion-
charge equilibration

Charge equilibration
(without fusion)

Light

Heavy

Bifurcation structure

_ Parameter Independent result (SLy4, Skm*)

_ Orientation independent with respect to the upper energy limit
(We have made collision deformed nuclei and compared them)

FUTURE STANDING PROBLEM AROUND

**Theoretical understanding of mass distribution is only at the beginning stage.
Indeed, we have shown only the discrepancy between fusion and charge equilibration, that is to say, the number of final fragment is one or not.**

>>> It means the difficulty of understanding the mass distribution (mass equilibration, mass transport) .